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Algebraic Topological Modeling for Cyberworld Design

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Abstract

The diversity of cyberworlds makes it hard to see consistency in terms of invariants. The consistency requires for us to abstract the most essentials out of the diversity, and hence the most abstract mathematics. It has been true in science in general, and in the theory of universe in particular. What are the most essential invariants in modeling cyberworlds? A branch of the most abstract mathematics is topology. For topology to be computable, it has to be algebraic. So, the searches have been for over two decades in algebraic topology for cyberworld invariants. Equivalence relations define invariants at various abstraction levels. The paper solely serves as an initial summary of algebraic topological resources for studying cyberworlds starting from the very elementary set theoretical level. High social impact application cases of e-financing and e-manufacturing are presented at the end.

0. Prologue: What are cyberworlds?

Cyberworlds are being formed in cyberspaces as computational spaces. My discovery of cyberworlds goes back to 1969 [4]. Now cyberspaces are on the web either intentionally or spontaneously, with or without design. Widespread and intensive local activities are melting each other on the web globally to create cyberworlds. The major key players of cyberworlds include e-finance that trades a GDP-equivalent a day and e-manufacturing that is transforming industrial production into Web shopping of product components and assembly factories. Without proper modeling, cyberworlds have continued to grow chaotic and are now out of human understanding and control.

A novel information model we named "an adjunction space model" serves to globally integrate local models. As an information model, it is also applicable to the category of irregular data models that capture spatio-temporal aspects of information worlds. Mathematically it is based on an incrementally modular abstraction hierarchy including cellular spatial structures in a homotopy theoretical framework [1, 2].

1. Set theoretical design

First of all, we start our design work of cyberworlds from defining a collection of objects we are looking at to construct them in cyberspaces. To be able to conduct automation on such collections by using computers as intelligent machines, each collection has to be a *set* because computers are built as set theoretical machines. Intuitively, a *set* X is a collection of all objects x having an identical *property*, say P(x). Symbolically $X = \{x \mid P(x)\}$. Any object in a set is called an element. A set without an element is named the *empty set* ϕ. A set is said *open* if all of its elements are interior. Given sets X and Y, computers perform s*et theoretical operations* such as the union $X \cup Y$, the intersection $X \cap Y$, the difference $X - Y$ (also denoted as $x \setminus y$), and the negation $\neg X$. Suppose we begin our cyberspace architecture design from a set X as the initial cyberspace. Given all elements u of an unknown cyberspace U, if they are confirmed to be the elements of our cyberspace X, the unknown cyberspace is called a *subset* of X or a subcyberspace of X and denoted as $U \subseteq X$. Thus, the subset check is automatically performed by processing $(\forall u)(u \in U \longrightarrow u \in X)$. The *closure* $\mathbb{U}$ of U is the intersection of all closed subsets of X, containing U. In other words, the closure $\mathbb{U}$ is the elements of X that are not the exterior elements of U. The set of all the subsets of X, $\{U \mid U \subseteq X\}$, is called a *power set* of X and denoted as 2^{x}. It is also called the *discrete topology* of X. The discrete topology is quite useful to design the cyberspace as consisting of subcyberspaces.

2. Topological design

Now, we go into the business of designing the cyberspace as the union of the subcyberspaces of X and their overlaps. The cyberspace thus designed is generally called a *topological space* (X, T) where $T \subseteq 2^{x}$. Designing a topological space is automated by the following specification:

1) $X \in T$ and $\varnothing \in T$;

2) For an arbitrary index set J,

$\forall\ j \in J\ (U_j \in T) \longrightarrow \bigcup_{j \in J} U_j \in T$;

3) $U, V \in T \longrightarrow U \cap V \in T$.

T is said to be the *topology* of the topological space (X, T). Given two topologies T_1 and T_2 on X such that $T_1 \subset T_2$, we say T_1 is *weaker* or *smaller* than T_2 (alternatively, we say that T_2 is *stronger* or *larger* than T_1. We also say T_2 is *finer* than T_1, or T_1 is *coarser* than T_2.). Obviously the *strongest topology* is the discrete topology (the power set) and the *weakest topology* is ∅. For simplicity, we often use X instead of (X, T) to represent a topological space whenever no ambiguity arises. When we see two topological spaces (X, T) and (Y, T'), how can we tell (X, T) and (Y, T') are equivalent? Here is a criterion for us to use computers to automatically validate that they are topologically equivalent. Two topological spaces (X, T) and (Y, T') are *topologically equivalent* (or *homeomorphic*) if there is a function $f : (X, T) \longrightarrow (Y, T')$ that is continuous, and its inverse exists and is continuous. We write $(X, T) \cong (Y, T')$ for (X, T) to be homeomorphic to (Y, T'). Then, how to validate the *continuity* of a function f? It amounts to check, first, $\forall\ B \in T', f^{\leftharpoonup} B \in T$, where $f^{\leftharpoonup} B$ means the inverse image of B by f, then, next, check the following:

B is open $\Leftrightarrow f^{-1}(B)$ is also open in X.

3. Functions

Given a function $f: X \rightarrow Y$, there are a total function and a partial function. For $f: X \rightarrow Y$ iff $\forall x \in X, \exists f(x)$, f is called a *total function*. A function $f: X' \rightarrow Y \mid X' \supseteq X$ is called a *partial function*, and not necessarily f(x) exists for every $x \in X$. For total functions, there are three basic types of relationships or mappings:

1. *Injective* or *into*, meaning $\forall x, y \in X\ x \neq y \Rightarrow f(x) \neq f(y)$; alternatively, $\forall x, y \in X\ f(x) = f(y) \Rightarrow x = y$;
2. *surjective* or *onto*, meaning $(\forall y \in Y)\ (\exists x \in X)\ [\, f(x) = y \,]$;
3. *bijective*, meaning injective and surjective.

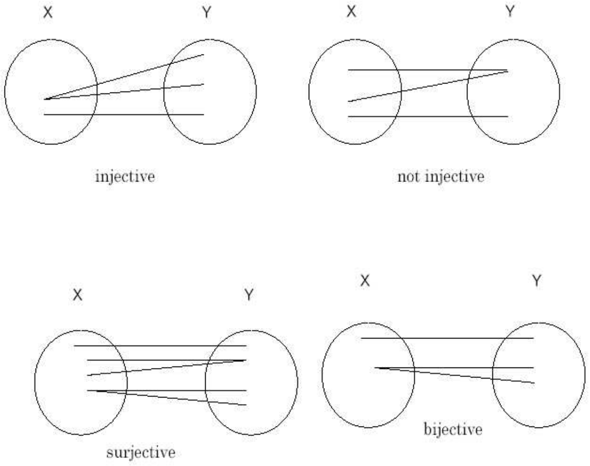

Figure 1. Functions.

4. Equivalence relations

For a binary relation $R \subseteq X \times X$ on a set X, R is :
reflexive if $(\forall x \in X)\ [xRx]$: reflexivity,
symmetric if $(\forall x, y \in X)\ [xRy \Rightarrow yRx]$:symmetry, and
transitive if $(\forall x, y, z \in X)\ [[xRy \Rightarrow yRz] \Rightarrow xRz]$: transitivity.

R is called an *equivalence relation* (in a notation ∼) if R is reflexive, symmetric and transitive.

Given $x \in X$, a subset of X defined by $x / \sim\ = \{y \in X: x \sim y\}$ is called the *equivalence class* of x. Here a class actually means a set; it is a tradition, and hard to be changed at this stage. The set of all the equivalence classes X/∼ is called the *quotient space* or the *identification space* of X.

$X / \sim\ = \{x / \sim\ \in 2^{X} \mid x \in X\} \subseteq 2^{X}$.

From the transitivity, for each $x \in X$, $x /\!\sim\ \neq \phi$, the followings hold:

$x \sim y \Leftrightarrow x/\!\sim\ = y/\!\sim$, and

$x \nsim y \Leftrightarrow x/\!\sim\ \cap\ y/\!\sim\ = \phi$.

This means a set X is *partitioned* (also called *decomposed*) into non-empty and *disjoint* equivalence classes.

Let us look at simple examples. In Euclidean geometry, given a set of figures, a *congruence relation* divides them into a disjoint union of the subsets of congruent figures as a quotient space; a *similarity relation* divides them into a disjoint union of the subsets of similar figures as a quotient space. Congruence and similarity relations are cases of affine transformations. A symmetry relation in group theory divides a set of figures into a disjoint union of the subsets of symmetric figures as a quotient space. In e-commerce, to be e-merchandise is an equivalence relation while e-trading is a poset (partially ordered) relation. In e-trading, a seller-buyer relation is asymmetric while an e-merchandise relation is symmetric because e-merchandise for sellers is also e-merchandise for buyers.

5. A quotient space (an identification space)

Let X be a topological space. Let f be a *surjective* (onto) and *continuous* mapping called a *quotient map* (often also called an *identification map*) that maps each point $x \in X$ to a subset (an equivalence class $x/\sim \in X/\sim$) containing x

$f: X \to X/\sim$.

Here, as explained before, "a map $f: X \to Y$ is surjective (onto)" means

$(\forall y \in Y)(\exists x \in X)[f(x) = y]$.

Suppose we take a surjective map f such that for subset X^0 of X, $X^0 \subseteq X$,
X^0 is open $\Leftrightarrow f^{-1}(X^0) \mid y \in A$ is open in X (this means f is continuous), $X/\sim$ is called a *quotient space* (or an *identification space*) by a *quotient map* (or an *identification map*) f. There is a reason why a *quotient space* is also called an *identification space*. It is because, as stated before, a quotient space is obtained by identifying each element (an equivalence class)

$x/\sim \in X/\sim$

with a point $x \in X$ that is contained in $x/\sim$.

6. An attaching space (an adjunction space, or an adjoining space)

Let us start with a topological space X and attach another topological space Y to it. Then,

$Y_f = Y \sqcup_f X = Y \sqcup X/\sim$

is an *attaching space* (an *adjunction space*, or an *adjoining space*) obtained by attaching (gluing, adjuncting, or adjoining) Y to X by an *attaching map* (an *adjunction map*, or an *adjoining map*) f (or by identifying each point $y \in Y_0 \mid Y_0 \subset Y$ with its image $f(y) \in X$ by a *continuous* map f) [3]. ⊔ denotes a disjoint union (another name is an "exclusive or") and often a + symbol is used instead.

Attaching map f is a *continuous* map such that

$f: Y_0 \to X$,

where $Y_0 \subset Y$. Thus, the attaching space $Y_f = Y \sqcup X/\sim$ is a case of *quotient spaces*

$Y \sqcup X/\sim = Y \sqcup_f X = Y \sqcup X/(x \sim f(y) \mid \forall y \in Y_0)$.

The *identification map* g in this case is

$g: Y \sqcup X \to Y \sqcup_f X = Y_f = Y \sqcup X/\sim = (Y \sqcup X - Y_0) \sqcup Y_0$.

7. Restriction and inclusion

For any function

$g: Y \to Z$

the *restriction* of g to X ($X \subseteq Y$) is:

$g|X = g \circ i: X \to Z$

where

$i: X \to Y$

is an *inclusion*, i.e.

$\forall x \in X, i(x) = x$.

8. Extensions and retractions of continuous maps

For topological spaces X and Y, and a subspace $A \subset X$, a continuous map $f: X \to Y$ such that $f|A: A \to Y$
is called a *continuous extension* (or simply an *extension*) of a map f|A from A onto X. An extension is, thus, a partial function.

A *restriction* r is a continuous extension of an identity map $1_A: A \to A$ onto X such that

$r: X \to A$.

Then, $r|A = 1_A$.

For $A \subset X$, A is called a *deformation retract* of X, denoted by $X \rightsquigarrow A$, if there is a *retraction* $r: X \to A$ such that $i \circ r \sim 1_X$.

If A is a single point $A = \{a\} \subset X$, A is called retractable and denoted by X *. $\rightsquigarrow$

9. Homotopy

Homotopy is a case of extensions. Let X and Y be topological spaces, $f, g: X \to Y$ be continuous maps, and $I = [1, 0]$. *Homotopy* is defined

$H: X \times I \to Y$

where for $t \in I$

$H = f$ when $t=0$, and

$H = g$ when $t=1$.

Homotopy is an extension of continuous maps

$H|X \times \{0\} = fi_0$, and

$H|X \times \{1\} = gi_1$

where

$i_0 = X \times \{0\} \to X$, and

$i_1 = X \times \{1\} \to X$.

Topological spaces X and Y are *homotopically equivalent* $X \simeq Y$, namely *of the same homotopy type*, if the following condition meets:
For two functions f and h

$f: X \to Y$ and $h: Y \to X$,

$h \circ f \simeq 1_X$ and $f \circ h \simeq 1_Y$,

where 1_X and 1_Y are identity maps

$1_X: X \to X$ and $1_Y: Y \to Y$.

Homotopy equivalence is more general than topology equivalence. Homotopy equivalence can identify a shape change that is topologically not any more equivalent after the change. While a shape element goes through deformation processes, the deformation processes are specified by a homotopy and validated by homotopy equivalence. As a matter of fact, from the viewpoint of the abstractness of invariance, homotopy equivalence is more abstract than set theoretical equivalence because, when we change a given set by adding or deleting elements, we can make the set homotopy equivalent by preserving the operation of add or delete and also the added or deleted elements.

10. Cellular structured spaces (cellular spaces)

A *cell* is a topological space X that is topologically equivalent (homeomorphic) to an arbitrary dimensional (say n-dimensional where n is a natural number) closed ball $\mathcal{B}^n$ called a closed *n-cell*. An open n-cell is denoted as Int $\mathcal{B}^n$ (also as $\mathring{\mathcal{B}}^n$ and more often as e^n). $\mathcal{B}^n$ is

$$\mathcal{B}^n = \{x \in \mathbb{R}^n, \|x\| \leq 1\},$$

namely a closed n-dimensional ball, and $\mathbb{R}^n$ is an n-dimensional real number.

$$\text{Int } \mathcal{B}^n = \mathring{\mathcal{B}}^n = \{x \in \mathbb{R}^n, \|x\| < 1\}$$

is an open n-dimensional ball and is an *interior* of $\mathcal{B}^n$).

$$\partial\mathcal{B}^n = \mathcal{B}^n - \mathring{\mathcal{B}}^n = S^{n-1}$$

is the *boundary* of $\mathcal{B}^n$, and it is an (n-1)-dimensional *sphere* S^{n-1}.

For a topological space X, a *characteristic map* $\mathcal{F}$ is a continuous function.

$$\mathcal{F}: \mathcal{B}^n \to X,$$

such that it is a homeomorphism:

$$\mathcal{F}: \mathring{\mathcal{B}}^n \longrightarrow \mathcal{F}(\mathring{\mathcal{B}}^n), \text{ and}$$

$$\mathcal{F}(\partial\mathcal{B}^n) = \mathcal{F}(\mathcal{B}^n) - \mathcal{F}(\mathring{\mathcal{B}}^n).$$

$e^n = \mathcal{F}(\mathring{\mathcal{B}}^n)$ is an *open n-cell*, and $\bar{e}^n = \mathcal{F}(\mathcal{B}^n)$ is a *closed n-cell*.

From a topological space X, we can compose a finite or infinite sequence of cells X^p that are subspaces of X, indexed by integer $\mathbb{Z}$, namely $\{ X^p \mid X^p \subseteq X, p \in \mathbb{Z}\}$ called a filtration, such that
X^p covers X (or X^p is a covering of X), namely,

$$X = \bigcup_{p \in \mathbb{Z}} X^p,$$

and X^p is a subspace of X, namely,

$$X^0 \subseteq X^1 \subseteq X^2 \subseteq \ldots \subseteq X^{p-1} \subseteq X^p \subseteq \ldots \subseteq X.$$

(this is called a *skeleton*). The skeleton with a dimension at most p is called a *p-skeleton*.

We also say that $C = \{X^p \mid X^p \subseteq X, p \in \mathbb{Z}\}$ is a *cell decomposition* of a topological space X, or a *partition* of a topological space X into subspaces X^p which are closed cells. (X, C) is called a *CW-complex*.

When we perform cell decomposition, by preserving cell attachment maps, we can turn cellular spaces into reusable resources. We name such preserved and shared information a *cellular database* and a system to manage it a *cellular database management system* (*cellular DBMS*).

To be more precise, according to J. H. C. Whitehead [5], given a topological space X, we *inductively* compose a *filtration* X^p with a *skeleton*

$$X^0 \subseteq X^1 \subseteq X^2 \subseteq \ldots \subseteq X^{p-1} \subseteq X^p \subseteq \ldots \subseteq X$$

as a topological space as follows:

(1) $X^0 \subset X$ is a subspace whose elements are 0-cells of X.

(2) X^p is composed from X^{p-1} by *attaching* (*adjuncting*, *adjoining*, or *gluing*) to it a disjoint union $\sqcup_i\mathcal{B}_i^p$ of closed p-dimensional balls via a surjective and continuous mapping called an attaching map (an adjunction map, an adjoining map, or a gluing map)

$$F: \sqcup_i\partial\mathcal{B}_i^p \to X^{p-1}.$$

In other words, we compose X^p from X^{p-1} by taking a disjoint union $X^{p-1} \sqcup (\sqcup_i\mathcal{B}_i^p)$ and by identifying each point x in $\partial\mathcal{B}_i^p$, $x \in \partial\mathcal{B}_i^p$, with its image $\mathcal{F}(x)$ by a continuous mapping

$$F_i = F \mid \partial\mathcal{B}_i^p : \partial\mathcal{B}_i^p \to X^{p-1}$$

such that $x \sim f_i(x)$ for each index i. Thus, X^p is a quotient space (the identification space)

$$X^p = X^{p-1} \sqcup (\sqcup_i\mathcal{B}_i^p) \mid (x \sim F_i(x) \mid \forall x \in \partial\mathcal{B}_i^p)$$

$$= X^{p-1} \sqcup_F (\sqcup_i\mathcal{B}_i^p)$$

and is a case of *attaching spaces* (*adjunction spaces* or *adjoining spaces*). The map F_i is a case of *attaching maps* (*adjunction maps*, *adjoining maps* or *gluing maps*) of a cell $\mathcal{B}_i^p$. A *filtration space* is a space homotopically equivalent to a filtration. The topological space X with the skeleton $X^0 \subseteq X^1 \subseteq X^2 \subseteq \ldots \subseteq X^{p-1} \subseteq X^p \subseteq \ldots \subseteq X$ is called a *CW-space*. As a cell complex, it is called a *CW-complex* as explained before.

We thus obtain a map $\mathcal{F}$ as a case of *identification maps*

$$\mathcal{F}: X^{p-1} \sqcup (\sqcup_i\mathcal{B}_i^p) \to X^{p-1} \sqcup_F (\sqcup_i\mathcal{B}_i^p) = X^p.$$

A characteristic map $\mathcal{F}$ for each n-cell $\mathcal{B}_i^p = \mathcal{F}(\mathcal{B}_i^p) \in X^p$ is

$$\mathcal{F}_i = \mathcal{F} \mid \partial\mathcal{B}_i^p : \partial\mathcal{B}_i^p \to X^{p-1}.$$

The embedding of X^{p-1} as a closed subspace of X^p is
$\mathcal{F} \mid X^{p-1} = X^{p-1} \to X^p$.

If a CW-space is diffeomorphic, it is equivalent to a *manifold* space.

11 An incrementally modular abstraction hierarchy

Although we do not go into the details, the considerations of abstraction levels explained so far for an incrementally modular abstraction hierarchy [8]. The adjunction spaces model the common properties of dominant commercial information systems being used by major private and public organizations by abstracting the common properties to be equivalent among different information systems as adjunction spaces, thus serving as a novel data model that can integrate information systems linearly and hence avoiding the combinatorial explosion of the integration workload. For automated linear interface generation after the linear integration at the adjunction space level, we use the *incrementally modular abstraction hierarchy* [8] as shown below such that we are interfaced to existing information systems to the extent we realize linear interoperability to perform the integrated system-wide tasks.

1. The homotopy level;
2. The set theoretical level;
3. The topological space level;
4. The adjunction space level;
5. The cellular space level;
6. The representation level;
7. The view level.

The details on this theme require intensive case analysis and case studies after careful theoretical studies. We are currently working on it with promising perspectives. The major problems we have been encountering are how to work with dominant existing systems that have no clean interoperability provisions. The relational model is a typical example.

12 Application cases

12.1 Web Information Modeling: What it is and what it is for?

Usually the business of Web information management systems is to manage information on the Web in close interaction with human cognition through information visualization via Web graphics [7]. The business of Web graphics is to project varieties of images on graphics screens for human understanding. Human understanding of displayed images is achieved by linking displayed images in the display space to human cognitive entities in the cognitive space. Often geometrically exact display misleads human cognition by the low priority geometrical shapes that are usually not the essential information in cyberworlds. Web graphics for Web information management has to deliver the essential messages on the screen for immediate human cognition at the speed to match the cyberworld changes [6].

Let us take a simple example.

12.2 Web Information Modeling of e-Finance

Suppose in e-finance a customer X has found the possibly profitable funds Y_0 posted on the Web at the home of a financial trading company Y during Web surfing as we do window-shopping for goodies. It is a Web window-shopping process and since the customer X and the trading company Y do not yet share the funds, X and Y are disjoint as denoted by $X \sqcup Y$. Let us also suppose for generality that X and Y are topological spaces. Since the funds Y_0 are a part of the properties of the financial trading company Y, $Y_0 \subseteq Y$ holds. The processes of e-financial trading on the Web as Web trading are represented on Web graphics as illustrated in Figure 2. Then, how the customer X is related to the trading company after the funds are identified for trading? The Web information model we present here precisely represents the relation by an *attaching map* f, and also represents the situation “the funds are identified for trading” as an *adjunction space* of two disjoint topological spaces X (the customer) and Y (the financial trading company), obtained by starting from the customer X and by attaching the financial trading company Y to the customer via a continuous function f by *identifying* each point $y \in Y_0 \mid Y_0 \subseteq Y$ with its image $f(y) \in X$ so that $x \sim f(y) \mid \forall\, y \in Y_0$. Thus, the *equivalence* denoted by $\sim$ plays the central role in Web information modeling to compose an adjunction space as the *adjunction space model* of Web information.

The adjunction space model illustrated above is quite essential and equally applied to e-manufacturing. It requires the exactly the identical technology to manage and display the e-manufacturing processes. For e-manufacturing to be effective to immediately meet market demands, it has to specify how varied sized components are assembled by a unified assembly design. By considering the e-financial trading presented so far as the assembly of the customers and the trading companies as the components of e-manufacturing, actually the Web information management systems and Web graphics technology for e-financial trading become applicable to e-manufacturing. Were we to use different technologies for different applications, fast growing cyberworlds could be neither managed by Web information management systems nor displayed on Web graphics in a timely manner.

12.3 Web Information Modeling of e-Manufacturing

Web information modeling of e-manufacturing by the adjunction space model is quite straightforward [7]. Basically, manufacturing on the Web called e-manufacturing is modeled as Web information consists of the following information on the e-manufacturing steps:

1) Product specification,
2) Assembly specification,
3) Parts shopping on the Web, and
4) Assembly site shopping on the Web.

Each step is decomposed into finer sub steps as needed. For example, the step 1 can be decomposed into:

1.1) Product market survey on the e-market,
1.2) Product requirement derivation from the survey, and
1.3) Product specification to meet the requirements.

The core of the whole Web technology for e-manufacturing is product and assembly modeling on the Web as Web information modeling. It is shown in Figure 3 using a simple assembly case of a chair with just two components of a seat and the support, for clear illustration of the most elementary assembly modeling. In e-manufacturing as an advanced manufacturing, the product components are defined to be modularly replaceable for higher quality components shopping, and also for most effective upgrades and repair.

It is clear that e-finance and e-manufacturing share the identical information modeling based on an adjunction space and equivalence.

13. Epilogue

Cyberworlds have been playing the central roles in the real world we live. Yet very little has been understood on them. What is happening is truly a type of the Genesis. The material presented here is in a hope to serve at least a minimum reference to understand cyberworlds.

14. Suggested readings

There is no book on cyberworld design *per se* yet. To make anything computable, there is algebra. To understand basic algebra, your will find the following useful:

“ Proofs and Fundamentals: A First Course in Abstract Mathematics” by Ethan D. Bloch (Birkhäuser, 2000).

Algebraic topology is essential to compute topological properties of cyberworlds and the followings are good textbooks:

1. “An Introduction to Topology and Homotopy” by Allan J. Sieradski (PWS-Kent Publishing Company, Boston, MA, USA, 1992). This is most recommended.
2. “A User’s Guide to Algebraic Topology” by C. T. J. Dodson and Philip E. Parker (Kluwer Academic Publication, Dordrecht, The Netherlands, 1997).
3. “Algebraic Topology” by Allen Hatcher (Cambridge University Press, Cambridge, UK, 2002); the up-to-date version can be downloaded freely from
   http://www.math.cornell.edu/~hatcher
   for personal use.

References

[1] Tosiyasu L. Kunii, “The Philosophy of Synthetic Worlds - Digital Genesis for Communicating Synthetic Worlds and the Real World -, in “Cyberworlds”, T. L. Kunii and A. Luciani (eds.), Springer-Verlag, pp. 5-15, (1998, Tokyo, Berlin, Heidelberg, New York).

[2] Tosiyasu L. Kunii and Hideko S. Kunii, “ A Cellular Model for Information Systems on the Web - Integrating Local and Global Information”, 1999 International Symposium on Database Applications in Non-Traditional Environments (DANTE'99), November 28-30, 1999, Heian Shrine, Kyoto, Japan, Organized by Research Project on Advanced Databases, in cooperation with Information Processing Society of Japan, ACM Japan, ACM SIGMOD Japan, pp. 19-24, IEEE Computer Society Press, Los Alamitos, California, U. S. A.

[3] Tosiyasu L. Kunii**,** Masumi Ibusuki**,** Galina Pasko, Alexander Pasko, Daisuke Terasaki and Hiroshi Hanaizumi, “Modeling of Conceptual Multiresolution Analysis by an Incrementally Modular Abstraction Hierarchy”, IEICE Transactions on Information and Systems, Vol. E86-D, No. 7, pp. 1181-1190, July 2003.

[4] Tosiyasu L. Kunii, “Invitation to System Sciences - Poetry, Philosophy and Science in Computer Age–”, (in Japanese), Journal of Mathematical Sciences, pp. 54-56 (October 1969), Science Publishing Co. Ltd., Tokyo, Japan.

[5] J. H. C. Whitehead,: “Algebraic Homotopy Theory”, *Proceedings of International Congress of Mathematics, II*, Harvard University Press, pp. 354-357, 1950.

[6] Tosiyasu L. Kunii, “Cyber Graphics”, Proceedings of the First International Symposium on Cyber Worlds (CW2002), November 6-8 2002 Tokyo, Japan, pp. 3-7, IEEE Computer Society Press, Los Alamitos, California, November 2002.

[7] Tosiyasu L. Kunii, “Web Information Modeling: The Adjunction Space Model”, Proceedings of the 2nd International Workshop on Databases in Networked Information Systems (DNIS 2002), pp. 58-63, The University of Aizu, Japan, December 16-18, 2002, Lecture Notes in Computer Science, Subhash Bhalla, Ed., Springer-Verlag, December, 2002.

[8] Tosiyasu L. Kunii, “What's Wrong with Wrapper Approaches in Modeling Information System Integration and Interoperability?”, Proceedings of the 3rd International Workshop on Databases in Networked Information Systems: User Interactions and Web Based Services, (DNIS 2003), September 22-24, 2003, The University of Aizu, Japan, Lecture Notes in Computer Science, Nadia Bianchi-Berthouze, Ed., pp. 86-96, Springer-Verlag, September, 2003.

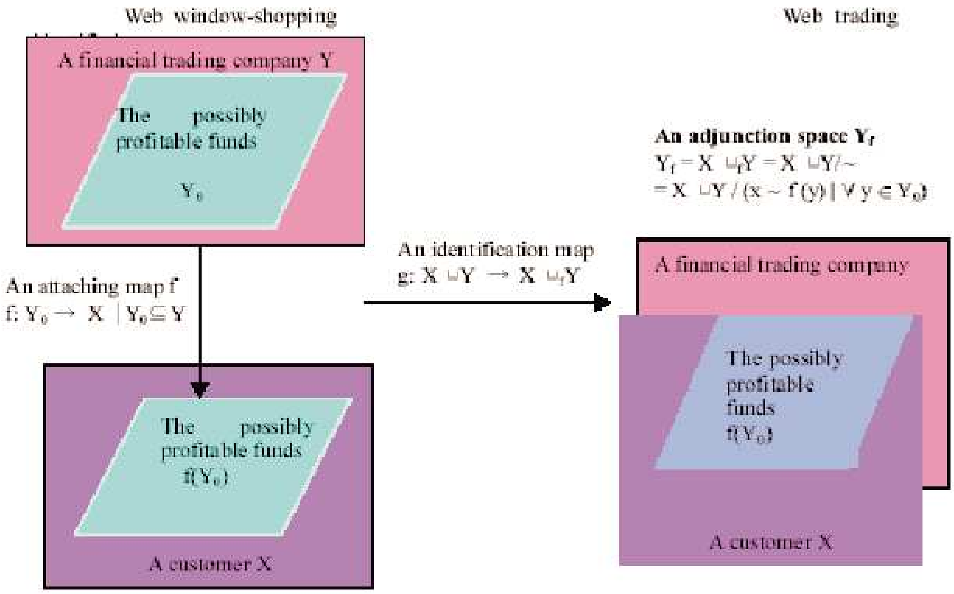

Figure 2. Financial trading processes on the Web displayed on Web graphics.

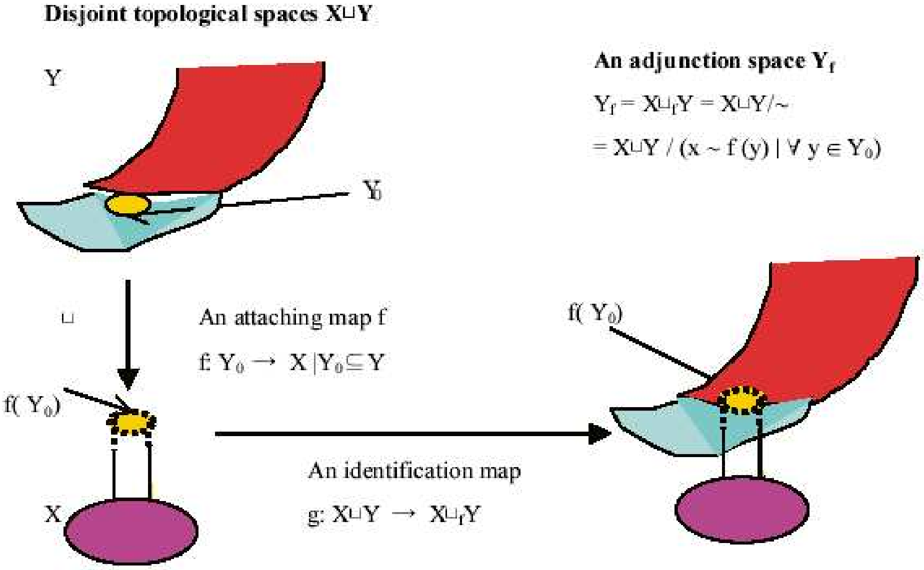

Figure 3. Web information modeling of e-manufacturing:
A case of modeling a chair assembly of the seat and the support.