Cyberworlds -Theory, Design and Potential-

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SUMMARY Cyberworlds are being formed in cyberspaces as computational spaces. Now cyberspaces are rapidly expanding on the Web either intentionally or spontaneously, with or without design. Widespread and intensive local activities are melting each other on the web globally to create cyberworlds. The major key players of cyberworlds include e-finance that trades a GDP-equivalent a day and e-manufacturing that is transforming industrial production into Web shopping of product components and assembly factories. Lacking proper theory and design, cyberworlds have continued to grow chaotic and are now out of human understanding and control. .This research first presents a generic theoretical framework and design based on algebraic topology, and also provides an axiomatic approach to theorize the potentials of cyberworlds.

Key words: *Social impacts of cyberworlds, algebraic topological theory of cyberworlds, axiomatizing cyberworlds, e-financing, e-manufacturing, e-commerce.*

1. What Are Cyberworlds?

Cyberworlds are being formed in cyberspaces as computational spaces. Now cyberspaces are rapidly expanding on the Web either intentionally or spontaneously, with or without design. [1, 2, 4, 9]. Widespread and intensive local activities are melting each other on the web globally to create cyberworlds. The diversity of cyberworlds makes it hard to see consistency in terms of *invariants*. The consistency requires for us to abstract the most essentials out of the diversity, and hence the most abstract mathematics. It has been true in science in general, and in the theory of physics to theorize the material worlds in particular.

What are the most essential *invariants* in theorizing cyberworlds? A branch of the most abstract mathematics is topology. For topology to be computable, it has to be algebraic. So, the researches have been conducted for over two decades on *cyberworld invariants* in algebraic topology. *Equivalence relations* define invariants at various abstraction levels.

The first half of the paper serves as an initial summary of algebraic topological resources for studying cyberworlds starting from the very elementary set theoretical level [9, 14]. High social impact application cases of e-financing and e-manufacturing are presented at the end of this part.

A novel information model we named "an adjunction space model" serves to globally integrate local models. As an information model, it is also applicable to the category of irregular data models that capture spatio-temporal aspects of information worlds. Mathematically it is based on an incrementally modular abstraction hierarchy including cellular spatial structures in a homotopy theoretical framework [1, 2].

Manuscript revised March 1, 2001.
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2. Set Theoretical Design

First of all, we start our design work of cyberworlds from defining a collection of objects we are looking at to construct them in cyberspaces. To be able to conduct automation on such collections by using computers as intelligent machines, each collection has to be a *set* because computers are built as set theoretical machines. Intuitively, a *set* X is a collection of all objects x having an identical *property*, say P(x). Symbolically $X = \{x \mid P(x)\}$. Any object in a set is called an element. A set without an element is named the *empty set* ϕ. A set is said *open* if all of its elements are interior. Given sets X and Y, computers perform s*et theoretical operations* such as the union $X \cup Y$, the intersection $X \cap Y$, the difference $X - Y$ (also denoted as $x \setminus y$), and the negation $\neg X$. Suppose we begin our cyberspace architecture design from a set X as the initial cyberspace. Given all elements u of an unknown cyberspace U, if they are confirmed to be the elements of our cyberspace X, the unknown cyberspace is called a *subset* of X or a subcyberspace of X and denoted as $U \subseteq X$. Thus, the subset check is automatically performed by processing $(\forall u)(u \in U \rightarrow u \in X)$. The *closure* $\bar{U}$ of U is the intersection of all closed subsets of X, containing U. In other words, the closure $\bar{U}$ is the elements of X that are not the exterior elements of U. The set of all the subsets of X, $\{U \mid U \subseteq X\}$, is called a *power set* of X and denoted as 2^X. It is also called the *discrete topology* of X. The discrete topology is quite useful to design the cyberspace as consisting of subcyberspaces.

3. Topological Design

Now, we go into the business of designing the cyberspace as the union of the subcyberspaces of X and their overlaps. The cyberspace thus designed is generally called a *topological space* (X, T) where $T \subseteq 2^{x}$. Designing a topological space is automated by the following specification:

1) $X \in T$ and $\varnothing \in T$;
2) For an arbitrary index set J,
$\forall j \in J\ (U_j \in T) \rightarrow \bigcup_{j \in J} U_j \in T$;
3) $U, V \in T \rightarrow U \cap V \in T$.

T is said to be the *topology* of the topological space (X, T). Given two topologies T_1 and T_2 on X such that $T_1 \subset T_2$, we say T_1 is *weaker* or *smaller* than T_2 (alternatively, we say that T_2 is *stronger* or *larger* than T_1. We also say T_2 is *finer* than T_1, or T_1 is *coarser* than T_2.). Obviously the *strongest topology* is the discrete topology (the power set) and the *weakest topology* is ∅. For simplicity, we often use X instead of (X, T) to represent a topological space whenever no ambiguity arises. When we see two topological spaces (X, T) and (Y, T'), how can we tell (X, T) and (Y, T') are equivalent? Here is a criterion for us to use computers to automatically validate that they are topologically equivalent. Two topological spaces (X, T) and (Y, T') are *topologically equivalent* (or *homeomorphic*) if there is a function $f : (X, T) \rightarrow (Y, T')$ that is continuous, and its inverse exists and is continuous. We write $(X, T) \cong (Y, T')$ for (X, T) to be homeomorphic to (Y, T'). Then, how to validate the *continuity* of a function f? It amounts to check, first, $\forall B \in T'$, $f^{\leftarrow} B \in T$, where $f^{\leftarrow} B$ means the inverse image of B by f, then, next, check the following:

B is open $\Leftrightarrow f^{-1}(B)$ is also open in X.

4. Functions

Given a function $f: X \rightarrow Y$, there are a total function and a partial function. For $f: X \rightarrow Y$ iff $\forall x \in X, \exists f(x)$, f is called a *total function*. A function $f: X' \rightarrow Y \mid X' \supseteq X$ is called a *partial function*, and not necessarily f(x) exists for every $x \in X$. For total functions, there are three basic types of relationships or mappings:

1. *Injective* or *into*, meaning $\forall x, y \in X\ x \neq y \Rightarrow f(x) \neq f(y)$; alternatively, $\forall x, y \in X\ f(x) = f(y) \Rightarrow x = y$;
2. *surjective* or *onto*, meaning $(\forall y \in Y)(\exists x \in X)$ $[f(x) = y]$;
3. *bijective*, meaning injective and surjective.

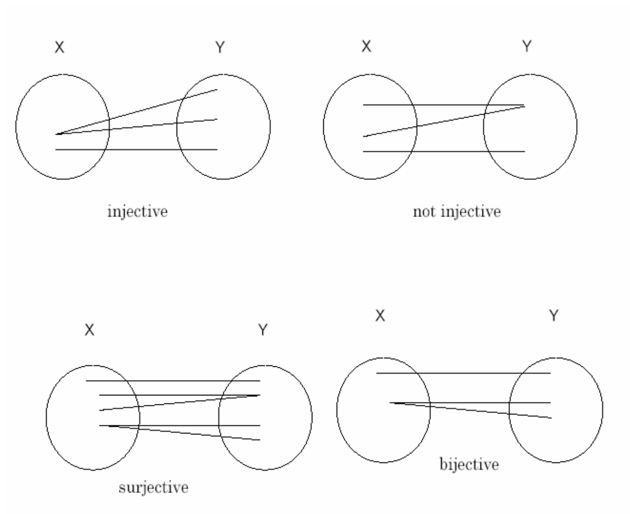

Figure 1. Functions.

5. Equivalence Relations

For a binary relation $R \subseteq X \times X$ on a set X, R is :
reflexive if $(\forall x \in X)$ [xRx]: reflexivity,
symmetric if $(\forall x, y \in X)$ [xRy ⇒ yRx]:symmetry, and
transitive if $(\forall x, y, z \in X)$ [[xRy ⇒ yRz] ⇒ xRz]:transitivity.

R is called an *equivalence relation* (in a notation ~) if R is reflexive, symmetric and transitive.

Given $x \in X$, a subset of X defined by $x / \sim = \{y \in X: x \sim y\}$ is called the *equivalence class* of x. Here a class actually means a set; it is a tradition, and hard to be changed at this stage. The set of all the equivalence classes X/~ is called the *quotient space* or the *identification space* of X.

$X / \sim = \{x / \sim \in 2^X \mid x \in X\} \subseteq 2^X$.

From the transitivity, for each $x \in X$, $x /\sim \neq \phi$, the followings hold:

$x \sim y \Leftrightarrow x /\sim = y /\sim$, and
$x \nsim y \Leftrightarrow x /\sim \cap y /\sim = \phi$.

This means a set X is *partitioned* (also called *decomposed*) into non-empty and *disjoint* equivalence classes.

Let us look at simple examples. In Euclidean geometry, given a set of figures, a *congruence relation* divides them into a disjoint union of the subsets of congruent figures as a quotient space; a *similarity relation* divides them into a disjoint union of the subsets of similar figures as a quotient space. Congruence and similarity relations are cases of affine transformations. A symmetry relation in group theory divides a set of figures into a disjoint union of the subsets of symmetric figures as a quotient space. In e-commerce, to be e-merchandise is an equivalence relation while e-trading is a poset (partially ordered) relation. In e-trading, a seller-buyer relation is asymmetric while an e-merchandise relation is symmetric because e-merchandise for sellers is also e-merchandise for buyers.

6. A Quotient Space (An Identification Space)

Let X be a topological space. Let f be a *surjective* (onto) and *continuous* mapping called a *quotient map* (often also called an *identification map*) that maps each point $x \in X$ to a subset (an equivalence class $x/\sim \in X/\sim$) containing x

$f: X \rightarrow X/\sim$.

Here, as explained before, "a map $f: X \rightarrow Y$ is surjective (onto)" means

$(\forall y \in Y)\ (\exists x \in X)\ [\ f(x) = y\ ]$.

Suppose we take a surjective map f such that for subset X^0 of X, $X^0 \subseteq X$,

X^0 is open $\Leftrightarrow f^{-1}(X^0) \mid y \in A$ is open in X (this means f is continuous), $X/\sim$ is called a *quotient space* (or an *identification space*) by a *quotient map* (or an *identification map*) f. There is a reason why a *quotient space* is also called an *identification space*. It is because, as stated before, a quotient space is obtained by identifying each element (an equivalence class)

$x/\sim \in X/\sim$

with a point $x \in X$ that is contained in $x/\sim$.

7. An Attaching Space (An Adjunction Space, or An Adjoining Space)

Let us start with a topological space X and attach another topological space Y to it. Then,

$Y_f = Y \sqcup_f X = Y \sqcup X/\sim$

is an *attaching space* (an *adjunction space*, or an *adjoining space*) obtained by attaching (gluing, adjuncting, or adjoining) Y to X by an *attaching map* (an *adjunction map*, or an *adjoining map*) f (or by identifying each point $y \in Y_0 \mid Y_0 \subset Y$ with its image $f(y) \in X$ by a *continuous* map f) [3]. $\sqcup$ denotes a disjoint union (another name is an "exclusive or") and often a + symbol is used instead.

Attaching map f is a *continuous* map such that

$f: Y_0 \rightarrow X$,

where $Y_0 \subset Y$. Thus, the attaching space $Y_f = Y \sqcup X/\sim$ is a case of *quotient spaces*

$Y \sqcup X/\sim\ = Y \sqcup_f X = Y \sqcup X/(x \sim f(y) \mid \forall y \in Y_0)$.

The *identification map* g in this case is

$g: Y \sqcup X \rightarrow Y \sqcup_f X = Y_f = Y \sqcup X/\sim\ = (Y \sqcup X - Y_0) \sqcup Y_0$.

8. Restriction and Inclusion

For any function

$g: Y \rightarrow Z$

the *restriction* of g to X ($X \subseteq Y$) is:

$g|X = g \circ i: X \rightarrow Z$

where

$i: X \rightarrow Y$

is an *inclusion*, i.e.

$\forall x \in X,\ i(x) = x$.

9. Extensions and Retractions of Continuous Maps

For topological spaces X and Y, and a subspace $A \subset X$, a continuous map $f: X \rightarrow Y$ such that $f|A: A \rightarrow Y$ is called a *continuous extension* (or simply an *extension*) of a map $f|A$ from A onto X. An extension is, thus, a partial function.

A *restriction* r is a continuous extension of an identity map $1_A: A \rightarrow A$ onto X such that

$r: X \rightarrow A$.

Then, $r|A = 1_A$.

For $A \subset X$, A is called a *deformation retract* of X, denoted by $X \rightsquigarrow A$, if there is a *retraction* $r: X \rightarrow A$ such that $i \circ r \sim 1_X$.

If A is a single point $A = \{a\} \subset X$, A is called retractable and denoted by $X \rightsquigarrow *$.

10. Homotopy

Homotopy is a case of extensions. Let X and Y be topological spaces, f, g: $X \rightarrow Y$ be continuous maps, and $I = [1, 0]$. *Homotopy* is defined

$H: X \times I \rightarrow Y$

where for $t \in I$

$H = f$ when $t=0$, and

$H = g$ when $t=1$.

Homotopy is an extension of continuous maps

$H|X \times \{0\} = fi_0$, and

$H|X \times \{1\} = gi_1$

where

$i_0 = X \times \{0\} \rightarrow X$, and

$i_1 = X \times \{1\} \rightarrow X$.

Topological spaces X and Y are *homotopically equivalent* $X \simeq Y$, namely *of the same homotopy type*, if the following condition meets:

For two functions f and h

$f: X \rightarrow Y$ and $h: Y \rightarrow X$,

$h \circ f \simeq 1_X$ and $f \circ h \simeq 1_Y$,

where 1_X and 1_Y are identity maps

$1_X: X \rightarrow X$ and $1_Y: Y \rightarrow Y$.

Homotopy equivalence is more general than topology equivalence. Homotopy equivalence can identify a shape change that is topologically not any more equivalent after the change. While a shape element goes through deformation processes, the deformation processes are specified by a homotopy and validated by homotopy equivalence. As a matter of fact, from the viewpoint of the abstractness of invariance, homotopy equivalence is more abstract than set theoretical equivalence because, when we change a given set by adding or deleting elements, we can make the set homotopy equivalent by preserving the operation of add or delete and also the added or deleted elements.

11. Cellular Structured Spaces (Cellular Spaces)

A *cell* is a topological space X that is topologically equivalent (homeomorphic) to an arbitrary dimensional (say n-dimensional where n is a natural number) closed ball $\mathcal{B}^n$ called a closed *n-cell*. An open n-cell is denoted as Int $\mathcal{B}^n$ (also as $\mathring{B}^n$ and more often as e^n). $\mathcal{B}^n$ is

$\mathcal{B}^n = \{x \in \mathbb{R}^n, \|x\| \leq 1\}$,

namely a closed n-dimensional ball, and $\mathbb{R}^n$ is an n-dimensional real number.

Int $\mathcal{B}^n = \mathring{B}^n = \{x \in \mathbb{R}^n, \|x\| < 1\}$

is an open n-dimensional ball and is an *interior* of $\mathcal{B}^n$).

$\partial\mathcal{B}^n = \mathcal{B}^n - \mathring{B}^n = S^{n-1}$

is the *boundary* of $\mathcal{B}^n$, and it is an (n-1)-dimensional *sphere* S^{n-1}.

For a topological space X, a *characteristic map* $\mathcal{F}$ is a continuous function.

$\mathcal{F}: \mathcal{B}^n \to X$,

such that it is a homeomorphism:

$\mathcal{F}: \mathring{B}^n \to \mathcal{F}(\mathring{B}^n)$, and

$\mathcal{F}(\partial\mathcal{B}^n) = \mathcal{F}(\mathcal{B}^n) - \mathcal{F}(\mathring{B}^n)$.

$e^n = \mathcal{F}(\mathring{B}^n)$ is an *open n-cell*, and $e^n = \mathcal{F}(\mathcal{B}^n)$ is a *closed n-cell*.

From a topological space X, we can compose a finite or infinite sequence of cells X^p that are subspaces of X, indexed by integer $\mathbb{Z}$, namely $\{ X^p \mid X^p \subseteq X, p \in \mathbb{Z}\}$ called a filtration, such that

X^p covers X (or X^p is a covering of X), namely,

$X = \bigcup_{p \in \mathbb{Z}} X^p$,

and X^p is a subspace of X, namely,

$X^0 \subseteq X^1 \subseteq X^2 \subseteq \ldots \subseteq X^{p-1} \subseteq X^p \subseteq \ldots \subseteq X$.

(this is called a *skeleton*). The skeleton with a dimension at most p is called a *p-skeleton*.

We also say that $C = \{X^p \mid X^p \subseteq X, p \in \mathbb{Z}\}$ is a *cell decomposition* of a topological space X, or a *partition* of a topological space X into subspaces X^p which are closed cells. (X, C) is called a *CW-complex*.

When we perform cell decomposition, by preserving cell attachment maps, we can turn cellular spaces into reusable resources. We name such preserved and shared information a *cellular database* and a system to manage it a *cellular database management system* (*cellular DBMS*).

To be more precise, according to J. H. C. Whitehead [5], given a topological space X, we *inductively* compose a *filtration* X^p with a *skeleton*

$X^0 \subseteq X^1 \subseteq X^2 \subseteq \ldots \subseteq X^{p-1} \subseteq X^p \subseteq \ldots \subseteq X$

as a topological space as follows:

(1) $X^0 \subset X$ is a subspace whose elements are 0-cells of X.

(2) X^p is composed from X^{p-1} by *attaching* (*adjuncting*, *adjoining*, or *gluing*) to it a disjoint union $\sqcup_i\mathcal{B}_i^p$ of closed p-dimensional balls via a surjective and continuous mapping called an attaching map (an adjunction map, an adjoining map, or a gluing map)

$F: \sqcup_i\partial\mathcal{B}_i^p \to X^{p-1}$.

In other words, we compose X^p from X^{p-1} by taking a disjoint union $X^{p-1}\sqcup (\sqcup_i\mathcal{B}_i^p)$ and by identifying each point x in $\partial\mathcal{B}_i^p$, $x \in \partial\mathcal{B}_i^p$, with its image $\mathcal{F}(x)$ by a continuous mapping

$F_i = F \mid \partial\mathcal{B}_i^p : \partial\mathcal{B}_i^p \to X^{p-1}$

such that $x \sim f_i(x)$ for each index i. Thus, X^p is a quotient space (the identification space)

$X^p = X^{p-1} \sqcup (\sqcup_i\mathcal{B}_i^p) \mid (x \sim F_i(x) \mid \forall x \in \partial\mathcal{B}_i^p)$

$= X^{p-1} \sqcup_F (\sqcup_i\mathcal{B}_i^p)$

and is a case of *attaching spaces* (*adjunction spaces* or *adjoining spaces*). The map F_i is a case of *attaching maps* (*adjunction maps*, *adjoining maps* or *gluing maps*) of a cell $\mathcal{B}_i^p$. A *filtration space* is a space homotopically equivalent to a filtration. The topological space X with the skeleton $X^0 \subseteq X^1 \subseteq X^2 \subseteq \ldots \subseteq X^{p-1} \subseteq X^p \subseteq \ldots \subseteq X$ is called a *CW-space*. As a cell complex, it is called a *CW-complex* as explained before.

We thus obtain a map $\mathcal{F}$ as a case of *identification maps*

$\mathcal{F}: X^{p-1} \sqcup (\sqcup_i\mathcal{B}_i^p) \to X^{p-1}\sqcup_F (\sqcup_i\mathcal{B}_i^p) = X^p$.

A characteristic map $\mathcal{F}$ for each n-cell $\mathcal{B}_i^p = \mathcal{F}(\mathcal{B}_i^p) \in X^p$ is

$\mathcal{F}_i = \mathcal{F} \mid \partial\mathcal{B}_i^p : \partial\mathcal{B}_i^p \to X^{p-1}$.

The embedding of X^{p-1} as a closed subspace of X^p is

$\mathcal{F} \mid X^{p-1} = X^{p-1} \to X^p$.

If a CW-space is diffeomorphic, it is equivalent to a *manifold* space.

12. An Incrementally Modular Abstraction Hierarchy

Although we do not go into the details, the considerations of abstraction levels explained so far for an incrementally modular abstraction hierarchy [8]. The adjunction spaces model the common properties of dominant commercial information systems being used by major private and public organizations by abstracting the common properties to be equivalent among different information systems as adjunction spaces, thus serving as a novel data model that can integrate information systems linearly and hence avoiding the combinatorial explosion of the integration workload. For automated linear interface generation after the linear integration at the adjunction space level, we use the *incrementally modular abstraction hierarchy* [8] as shown below such that we are interfaced to existing information systems to the extent we realize linear interoperability to perform the integrated system-wide tasks.

1. The homotopy level;
2. The set theoretical level;
3. The topological space level;
4. The adjunction space level;
5. The cellular space level;
6. The representation level;
7. The view level.

The details on this theme require intensive case analysis and case studies after careful theoretical studies. We are currently working on it with promising perspectives. The major problems we have been encountering are how to work with dominant existing systems that have no clean interoperability provisions. The relational model is a typical example.

13. Application Cases

13.1 Web Information Modeling: What It Is and What It Is for?

Usually the business of Web information management systems is to manage information on the Web in close interaction with human cognition through information visualization via Web graphics [7]. The business of Web graphics is to project varieties of images on graphics screens for human understanding. Human understanding of displayed images is achieved by linking displayed images in the display space to human cognitive entities in the cognitive space. Often geometrically exact display misleads human cognition by the low priority geometrical shapes that are usually not the essential information in cyberworlds. Web graphics for Web information management has to deliver the essential messages on the screen for immediate human cognition at the speed to match the cyberworld changes [6].
Let us take a simple example.

13.2 Web Information Modeling of e-Finance

Suppose in e-finance a customer X has found the possibly profitable funds Y_0 posted on the Web at the home of a financial trading company Y during Web surfing as we do window-shopping for goodies. It is a Web window-shopping process and since the customer X and the trading company Y do not yet share the funds, X and Y are disjoint as denoted by $X \sqcup Y$. Let us also suppose for generality that X and Y are topological spaces. Since the funds Y_0 are a part of the properties of the financial trading company Y, $Y_0 \subseteq Y$ holds. The processes of e-financial trading on the Web as Web trading are represented on Web graphics as illustrated in Figure 2. Then, how the customer X is related to the trading company after the funds are identified for trading? The Web information model we present here precisely represents the relation by an *attaching map* f, and also represents the situation "the funds are identified for trading" as an *adjunction space* of two disjoint topological spaces X (the customer) and Y (the financial trading company), obtained by starting from the customer X and by attaching the financial trading company Y to the customer via a continuous function f by *identifying* each point $y \in Y_0 \mid Y_0 \subseteq Y$ with its image $f(y) \in X$ so that $x \sim f(y) \mid \forall\, y \in Y_0$. Thus, the *equivalence* denoted by ~ plays the central role in Web information modeling to compose an adjunction space as the *adjunction space model* of Web information.

The adjunction space model illustrated above is quite essential and equally applied to e-manufacturing. It requires the exactly the identical technology to manage and display the e-manufacturing processes. For e-manufacturing to be effective to immediately meet market demands, it has to specify how varied sized components are assembled by a unified assembly design. By considering the e-financial trading presented so far as the assembly of the customers and the trading companies as the components of e-manufacturing, actually the Web information management systems and Web graphics technology for e-financial trading become applicable to e-manufacturing. Were we to use different technologies for different applications, fast growing cyberworlds could be neither managed by Web information management systems nor displayed on Web graphics in a timely manner.

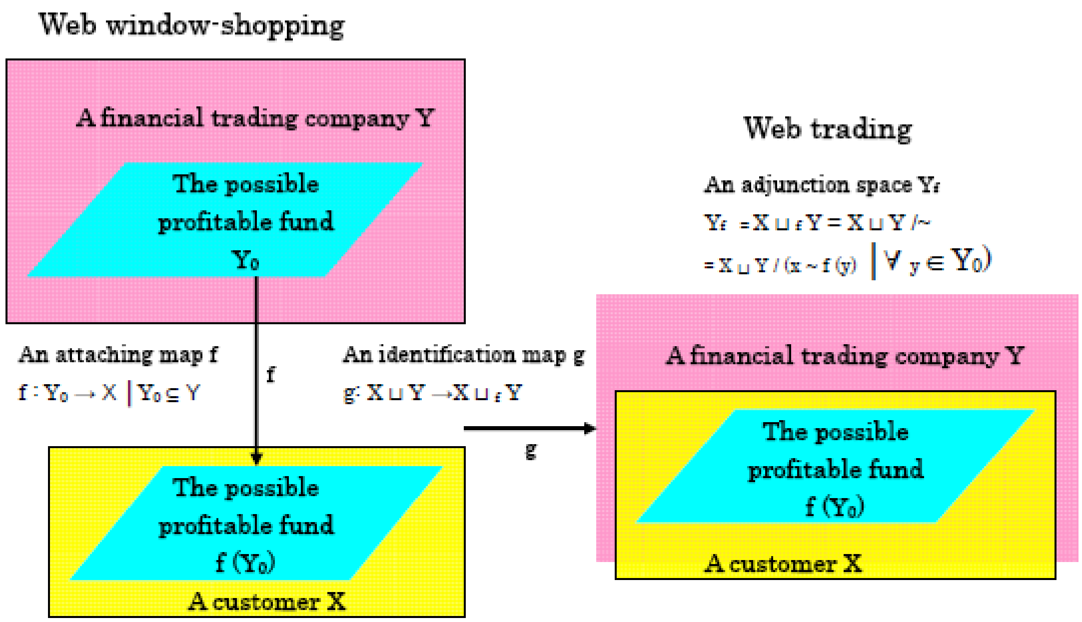

Figure 2. Financial trading processes on the Web displayed on Web graphics.

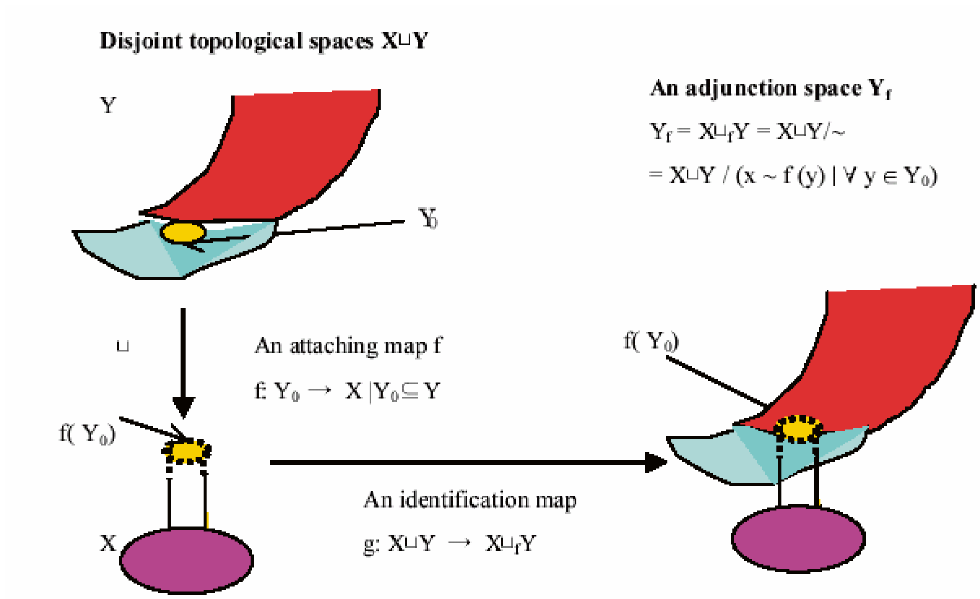

Figure 3. Web information modeling of e-manufacturing:
A case of modeling a chair assembly of the seat and the support.

13.3 Web Information Modeling of e-Manufacturing

Web information modeling of e-manufacturing by the adjunction space model is quite straightforward [7]. Basically, manufacturing on the Web called e-manufacturing is modeled as Web information consists of the following information on the e-manufacturing steps:

1) Product specification,
2) Assembly specification,
3) Parts shopping on the Web, and
4) Assembly site shopping on the Web.

Each step is decomposed into finer sub steps as needed. For example, the step 1 can be decomposed into:

- 1.1) Product market survey on the e-market,
- 1.2) Product requirement derivation from the survey, and
- 1.3) Product specification to meet the requirements.

The core of the whole Web technology for e-manufacturing is product and assembly modeling on the Web as Web information modeling. It is shown in Figure 3 using a simple assembly case of a chair with just two components of a seat and the support, for clear illustration of the most elementary assembly modeling. In e-manufacturing as an advanced manufacturing, the product components are defined to be modularly replaceable for higher quality components shopping, and also for most effective upgrades and repair.

It is clear that e-finance and e-manufacturing share the identical information modeling based on an adjunction space and equivalence.

14. An Axiomatization of Cyberworlds - The First Order Abstraction -

As we have finished to define cyberworlds in cyberspaces in algebraic topology [9, 14], we axiomatize cyberworlds as the next part of the research [28]. Any axiomatization to be meaningful, we have to go back to the past experiences to abstract the core hypotheses as a set of axioms.

My experience of discovering cyberworlds goes back to 1969 [4]. A proposal to study cyberworlds as an academic discipline was filed to the government creating Information Science Laboratory at the Faculty of Science of the University of Tokyo in 1970 with Graduate Course of Information Science. In 1975 it was upgraded to Information Science Department. Cyberworlds are closely related to the real world we live, intentionally or unintentionally. In certain areas, they have grown in their scales far beyond those of the real world. For example, in financial trading, a daily trading in the cyberworlds of e-trading in its amount is far beyond that of GDP.

The work to be presented here is a progress report and interim to clarify the reasons of such fast growth of cyberworlds and to mature the clarification to the level high enough to make it an academic discipline of cyberworlds. In establishing any academic disciplines at the level of those of exact sciences such as mathematics, physics and chemistry, we have to first axiomatize (or hypothesize) cyberworlds, derive theorems (or theories), and prove that they meet the axioms. Euclid of Alexandria has done it on geometry in around 300 BC [10].

In axiomatizing cyberworlds, we rely on the knowledge of well known worlds. One of the well known worlds, actually the best known, is the real world we live. The mere investigation of what have happened in the real world brings us into cosmology. Cyberspaces in which cyberworlds are created span on the artificials of networked computers. It means we can safely limit our first scope to the real world inside the period of the human history, and then extend it later as needed.

Historians do not axiomatize the human history, and they mainly record and analyze it around the rise and fall of the great powers as the indices. Let us suppose we measure the indices by two parameters: 1. the *power areas*, and 2. the *power periods*. In setting the parameter values of extremely complex systems as the human history, following the successful disciplines of exact sciences, we start from the first order approximation. The history as a whole does provide enough data for abstracting the first order approximation of the axioms on the power areas and the power periods [17, 18]. For simplicity, let us name it as the *first order abstraction*.

The great powers generally mean military, economical, political, cultural, and/or religious domination. Globally, the world is actually nonlinear. An example is seen in the shift of the cosmic view from the Ptolemaic theory to the Copernican theory. This is a good example of a shift to a globally correct approximate world model from a locally correct globally wrong world model. We will see later as the conclusion, that the cyberworlds drive the real world in its power and area into nonlinearity.

Questions on the validity of Paul Kennedy's prediction on the world power shift arose when I read his famous book, "The Rise and Fall of the Great Powers" [16], immediately after its publication in 1987. First of all, the time period he considered, namely from 1500 to 2,000, looked too short to make any valid prediction of the future history. Hence, his prediction of the rise of Japan as the great power after the USA that succeeded the great power of England seemed unrealistic. Here were the 1st order approximation hypotheses of the great power shift I counter proposed the next year in 1988 [12] and also in 1989 [13] with a proof to invalidate the Paul Kennedy's prediction.

Axioms

Axiom 1 The *power* area *size* (namely, the size of the major area of a given great power) is in proportion to the information speed (namely, the speed of the information made available to the power).

Axiom 2 The *power period* is in inverse proportion to the information speed.

The *Time Period* Considered: From the Egyptian Dynasties to the current world, namely from 3,100 BC to 2,000 AD.

The *Initial Power Area:* The Mediterranean Sea and the surroundings containing Cairo, Athens and Rome with the size of about 2 million km^2.

Egyptian Dynasties

3,100 BC: The union of Upper and Lower Egypt by Menes.

332 BC: The acquisition of Egypt by Alexander the Great of Macedon.

Roman Empire (Pax Romana)

27 BC: Octavian became Emperor Augustus.

476 AD: The last king of the Western Roman Empire deposed by the German King Odoacer.

1453 AD: The end of the Eastern Roman Empire.

Information speed was 5 km/hour. Just before the battle of Marathon in 490 BC, Pheidippides ran 241 km (150 miles) from Athens to Sparta in two days. The validation of the model is against Pax Britanica and Pax Americana. And then, we can predict the nature of the current and future powers (if any!).

Pax Britanica

As the preliminaries of Pax Britanica, there was the age of great voyages 1400 - 1650. Pax Britanica took place as an epoch making social change known as the *industrial revolution*: British change from agricultural to industrial economies, took place during 1750 - 1850, and then spread out to the Continental Europe and the USA covering the Atlantic cities, London, Paris and New York, with an area size of the order of 20 million km^2. The core of the industrial revolution was founded on the British engineering invention of a series of steam engines as typically seen in British engineer Thomas Savery's invention of high pressure steam engine in 1698, and the improvement to the current reliable design with a separate steam condenser and also with the crank and crosshead mechanism by another British engineer James Watt in 1769. From early 1800 steam engine ships and from 1829 locomotives built by a British engineer George Stephenson became popular. In 1829, the Rocket locomotive carried passengers at a speed of 36 mph (58 km/hour).

Pax Americana

The key action taken as the preliminaries to initiate Pax Americana was symbolized in a slogan "to advance knowledge." The real core of the action was the establishment of "research universities" in the USA. In 1990, the formation of the Association of American Universities signified the growth of American research universities during the years of 1900 - 1940, in terms of the numbers of PhDs produced, the volumes in the libraries and dollars expended for research.

The first power symbol of Pax Americana was an aircraft. The Wright Brothers made the first powered and controlled flight in 1903 in North Carolina, and for 45 minutes in 1907. The World War I, 1914 - 1918, saw the beginning of the use of air crafts for wars. In 1924 Imperial Airways in UK gave a birth to a commercial air route. In the World War II, 1939 - 1945, air forces were first intensively used. 1954 Boeing 707 was the first popularly used jet passenger aircraft. Usual speed of passenger flights is now close to 1000 km/hour connecting the Pacific Rim cities, such as San Francisco, Tokyo, Peking, Seoul, Taipei, Hong Kong, Singapore and Sydney, in the area of 12,000 million km^2 as a part of the worldwide networks of commercial air routes. Note that the whole globe surface is 50,000 million km^2.

The computer industry was the second power symbol of Pax Americana, and still is. Here is a brief chronological sketch.

1930-40: The Turing Machine and computability theory were developed by British mathematician Alan Turing in 1937. This is known as Alan Turing's mathematical abstraction of computability.

1943-46: Vacuum tube-based ENIAC was built at Moore School of Electrical Engineering of the University of Pennsylvania by John Mauchly and J. P. Eckert.

1948: William Bradfield Shockley invented transistors at Bell Telephone Laboratories.

1964: IBM 360 dominance of mainframes started.
Mid 1970: UNIX by Dennis Ritchie and Kenneth Thompson at Bell Laboratories initiated the emergence of minicomputers and workstations.
1980: Patterson and Ditley at the University of California, Berkeley invented RISC.
1987: SPARC architecture machine by Sun Microsystems, a derivative of RISC II machines of Patterson and Ditley, have taken 58.8 % share in the workstation market in 1991.
1990-: The Intel and Microsoft dominance of PC (personal computer) market share has been leading the world computer industry.

Summary of the Great Power Shift from 3,100 BC to 1987 AD is shown in Table 1.

The Great Powers	Information Carrier	Information Speed	The Power Area Size	The Power Period
Pax Romana	human feet networks	5 - 10 km/hour	2 million km^2	1000 years
Pax Britanica	surface vehicle networks	50 - 100 km/hour	20 million km^2	100 years
Pax Americana	aircraft networks	500- 1000 km/hour	200 million km^2 (40 % of the whole globe surface)	10 years

Tab. 1 Summary of the Great Power Shift from 3,100 BC to 1987 AD.

The axioms are validated as the first order abstraction of the human history..

Pax Informatica as the cyberworld era

Crash: An Economic Crisis and Chaos

The closing of the gold window by Richard M. Nixon on Sunday, August 15, 1971 had laid the ground for monetary crisis around the world [11]. The gold window was established in 1946 based on the Bretton Woods Agreement Act, prepared by the representatives of major trading nations, met in 1944 in Bretton Woods, New Hampshire, and signed by President Harry S. Truman on Tuesday, July 31, 1945.

Even with a strong economy, on October 19, 1987 called the Black Monday and the next day, October 20, 1987 called the Terrible Tuesday, the entire financial system of the USA came close to a complete meltdown. Computer networks have linked stock exchanges around the world into one market place as cyberworlds.

Now, let us add the great power shift from 1988 AD and beyond to the Table 2.

Theorem as Prediction:

Nonlinear, and quality dominate the world power shift in the cyberworld era.

Since the axiom were validated, how about and what about the prediction of the future of the world? Unlike Paul Kennedy's prediction, X is not Japonica. Now, for the first time in the human history, it is not the quantity but the quality that takes the lead and will be the master of the stage and scenes of the real world. Computer networks have linked the world at the

The Great Powers	Information Carrier	Information Speed	The Power Area Size	The Power Period
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Pax Americana	aircraft networks	500- 1000 km/hour	200 million km^2 (40 % of the whole globe surface)	10 years
Pax Informatica	Computer networks	0.5 billion km/hour	500 thousand times of the whole globe surface	5 minutes

Tab. 2 The great power shift from 1988 AD and beyond.

information' speed enough for any power to have a power area size far beyond the whole area of the globe, but with a momentary power period making the global world economically unstable as Soros has pointed out [12]. Through the digital TV technology the USA and Europe have been developing, TV and computer networks can merge together. Computers have become more than electronic theaters. Computer networks can broadcast computer-simulated scenes of the "nuclear winter" presented by Carl Sagan of Cornell University in 1983, forcing the power shift from military to economic dominance. The military use of the Prometheus' nucleic fire is now blocked. The power period of 5 minutes causes fast switching of momentary great powers, forcing the great powers to turn into cooperative powers. It means no more rise and fall of the great powers. Hence, Pax Japonica is impossible. Japan will be one of the cooperative world powers. The world architectural model has to be changed from a monolithic linear world power shift model such as

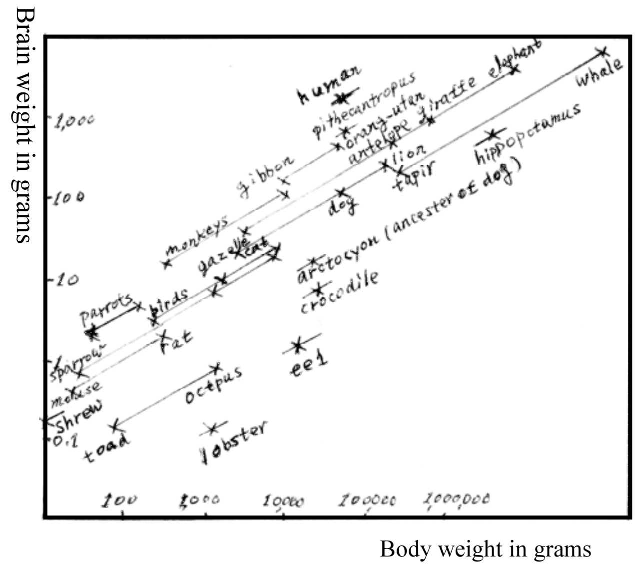
Pax Romana to a cellular structured space model or, when diffeomorphism holds, a manifold model, where varieties of coordinates called "cells" or "charts" coexist [13-15]. The coordinates can represent any values, such as economical, military, cultural, religious, humane or even ethical values. The excess power of computer networks allows the world to select the expert goals from the trivial. Even old analogue TV networks have already caused world power meltdowns. The domino effect of world power meltdowns took place in the Eastern Europe in Romania in 1989, in Germany in 1990, and finally in the Soviet Union in the late 1991. Computer networks can cause further extreme power meltdowns. Media events or media shows in the digital cyberspaces of synthetic worlds that I named virtual worlds in 1984 [19, 20] will play the central roles in real world decision making in the worlds of politics, economics, industry and commerce. Electronic commerce (EC) is an example. We are shifting to Pax Informatica that is not any more another great power, but a cooperative power.

The theorems and lemma as the prediction of the future of the real world are derived from the axioms.

Theorem 1 on the power area of cyberworlds:
500 thousand times of the whole globe surface.
Theorem 2 on the power period of cyberworlds:
5 minutes.
Lemma
For the first time in human history, the thousands of years old and linear great **power architecture** is going to fade out, and **nonlinear and cooperative power architecture** supported by digital and interactive networks is coming in.

15. The 3rd Axiomatization of the Evolution of Cyberworlds Also as the First Order Abstraction

Axiomatizing the *evolution of cyberworlds* as the first order abstraction require us to look into the evolution of life going beyond the human history. According to serious researches on the nature of the evolution of life, it is proved that there are two types: 1. The *dead ended evolution*, and 2. the *progressive evolution*, as explained in the book by Paul Chauchard [16]. It is derived from long



Tab. 3 Lapicque table [21 (page 55)]

researches on the changes in relative dimensions of parts of the body that are correlated with the changes in overall body size, named *allometry*, by Julian S. Huxley and Georges Teissier in 1936 [22–24]. Chauchard has named a tabular representation of the evolution of life by allometry, taking the brain as the key evolutionary part of the body, "*Lapicque table*" (Table 3) to dedicate its contribution to Louis Lapicque, his professor [21].

The evolution of a specific part of the body to adapt to environmental changes including competitions among species for survival, generally ends up in losing the flexibility to continue to adapt to further big environmental changes as typically known in the case of the extinction of dinosaurs and mammoths. This type of evolutions is known to be the type 1 *dead ended evolution*.

Certainly the brain as a soft organ, does not provide any physical function, even hammering, attacking or warming up the body. Still its function works to borrow and utilize any of such functions anytime when needed any amount as needed. The relatively well developed brain has won in its power to adapt to the competitive environmental changes over the development of the other organs with any physically specialized functions. When I was exposed to the Lapicque table through the book entitled "Précis de Biologie Humaine – Les Bases Organiques du Compotement et de la Pensée -" by Paul Chauchard [21] soon after the graduation from a high school in 1957, it has changed my life view. After 47 years, here is one tiny academic outcome from my side.

Cyberworlds and the real world have their evolutional functions similar to those in the evolution of life. Unlike the real world, cyberworlds lack direct physical power. Yet it can control direct drive motors to perform abundant physical movements flexibly. Actually by borrowing any functions from the real world at the light speed, cyberworlds are performing more and more drastic physical, logical and semi-intelligent functions in the real world. Hence, we can safely derive the fist order abstraction of the axiom of the evolutional power of cyberworlds:

Axiom 3 In the evolutionary power, borrowing and utilizing functions exceed owned functions.

As a matter of fact, for example, on the web, the cyberworlds of GPL-based open sources have more potentials to adapt to the rapidly changing computing applications than closed proprietary software because of the GPL to borrow and utilize functions mutually and returning the results to open sources according to the GPL [26, 27]. The potentials of GPL-based open source software in education to develop IT professionals in exploding population areas on the earth will save the human future in overcoming the critical shortage of IT professionals in developing fundamental software such as embedded OS and real time controllers which can never been successfully developed on top of the proprietary and closed OS. It is simply because, unlike developing application software running on the basic system software, developing basic system software itself requires in-detail knowledge of the core software and hardware functionality such as interrupt mechanisms, scheduling mechanisms, queue handling, device drivers, input/output interfacing, and storage structures. A certain experience-based note was presented without any axioms in 2001 [25].

The following theorem is derived from Axiom 3.

Theorem on limited resource securing wars:

Wars fighting for limited resources on the earth such as land and oil do not pay compared to the peace to enjoy sharing unlimited knowledge of open source software (and hardware) in cyberworlds.

The proof is left to the readers to enjoy. To make my point clear, the use of open source software simply to cut down the software developing cost is only an economical short term decision, and does not belong to my discussions here. In any event, it is a questionable decision because GPL requests the resulting software to be open, and hence if the decision was made purely in a short term based on a short term cost and profit expectation, the real outcome in a long term will not be what expected. The current situation in open source software is thus in need of Axiom 3 to firmly understand its essentials. For daily use, particularly for desktops, there is no doubt for casual users proprietary OS are easier because of established vender support with a reasonable cost. Open source software for daily and casual desktop use still waits for a reasonable business model to provide good enough integration, installation and maintenance. It certainly is a part of cyberworld evolution models, and is going to be presented elsewhere from the viewpoint of the evolution of life.

16. Suggested Textbooks of Algebraic Topology

There is no book on the science of cyberworlds *per se* yet. To make cyberworlds computable, there is algebra. Algebraic topology is essential to compute topological properties of cyberworlds and the followings are good textbooks:

1. "An Introduction to Topology and Homotopy" by Allan J. Sieradski (PWS-Kent Publishing Company, Boston, MA, USA, 1992). This is most recommended.
2. "A User's Guide to Algebraic Topology" by C. T. J. Dodson and Philip E. Parker (Kluwer Academic Publication, Dordrecht, The Netherlands, 1997).
3. "Algebraic Topology" by Allen Hatcher (Cambridge University Press, Cambridge, UK, 2002); the up-to-date version can be downloaded freely from
   http://www.math.cornell.edu/~hatcher
   for personal use.

17. Conclusions

We have shown cyberworlds to be theorized formally by algebraic topology. After that, the later part of the research is dedicated to the axiomatization of cyberworlds and derivation of the theorems. The axiomatic approach is pursued in search of more rigor and firmness. Increasing impact of cyberworlds demands the firm academic discipline to be constructed to make cyberworlds better understood and possibly to prevent negative side effects. After 36 years of research and education, the stage to realize it is still at its dawn. Cyberworlds have been grown in their core without consistency lacking theory and design principles , and hence are mostly unknown in their nature. It makes the axiomatization closer to hypothesizing in experimental physics rather than that in pure mathematics. In their scale, cyberworlds are ,at least in financial trading, at the level such that activities in a day in cyberworlds exceed those in a year in the real world, forcing us to better understand cyberworlds as a discipline rather than as a collection of phenomena.

References

[1] Tosiyasu L. Kunii, "The Philosophy of Synthetic Worlds - Digital Genesis for Communicating Synthetic Worlds and the Real World -, in "Cyberworlds", T. L. Kunii and A. Luciani

(eds.), Springer-Verlag, pp. 5-15, (1998, Tokyo, Berlin, Heidelberg, New York).

[2] Tosiyasu L. Kunii and Hideko S. Kunii, " A Cellular Model for Information Systems on the Web - Integrating Local and Global Information", 1999 International Symposium on Database Applications in Non-Traditional Environments (DANTE'99), November 28-30, 1999, Heian Shrine, Kyoto, Japan, Organized by Research Project on Advanced Databases, in cooperation with Information Processing Society of Japan, ACM Japan, ACM SIGMOD Japan, pp. 19-24, IEEE Computer Society Press, Los Alamitos, California, U. S. A.

[3] Tosiyasu L. Kunii**,** Masumi Ibusuki**,** Galina Pasko, Alexander Pasko, Daisuke Terasaki and Hiroshi Hanaizumi, "Modeling of Conceptual Multiresolution Analysis by an Incrementally Modular Abstraction Hierarchy", IEICE Transactions on Information and Systems, Vol. E86-D, No. 7, pp. 1181-1190, July 2003.

[4] Tosiyasu L. Kunii, "Invitation to System Sciences - Poetry, Philosophy and Science in Computer Age–", (in Japanese), Journal of Mathematical Sciences, pp. 54-56 (October 1969), Science Publishing Co. Ltd., Tokyo, Japan.

[5] J. H. C. Whitehead,: "Algebraic Homotopy Theory", *Proceedings of International Congress of Mathematics, II*, Harvard University Press, pp. 354-357, 1950.

[6] Tosiyasu L. Kunii, "Cyber Graphics", Proceedings of the First International Symposium on Cyber Worlds (CW2002), November 6-8 2002 Tokyo, Japan, pp. 3-7, IEEE Computer Society Press, Los Alamitos, California, November 2002.

[7] Tosiyasu L. Kunii, "Web Information Modeling: The Adjunction Space Model", Proceedings of the 2nd International Workshop on Databases in Networked Information Systems (DNIS 2002), pp. 58-63, The University of Aizu, Japan, December 16-18, 2002, Lecture Notes in Computer Science, Subhash Bhalla, Ed., Springer-Verlag, December, 2002.

[8] Tosiyasu L. Kunii, "What's Wrong with Wrapper Approaches in Modeling Information System Integration and Interoperability?", Proceedings of the 3rd International Workshop on Databases in Networked Information Systems: User Interactions and Web Based Services, (DNIS 2003), September 22-24, 2003, The University of Aizu, Japan, Lecture Notes in Computer Science, Nadia Bianchi-Berthouze, Ed., in press, Springer-Verlag, September, 2003.

[9] Tosiyasu L. Kunii, "Cellular Model", Research Memo, Oct, 1999.

[10] Glenn R. Morrow , "Proclus: A Commentary on the First Book of Euclid's Elements", , Princeton University Press, 1992.

[11] Joel Kurtzman, "The Death of Money", Simon & Schuster, New York, NY, 1993.

[12] George Soros, "The Crisis of Global Capitalism - Open Society Endangered-", Public Affairs, New York, 1998.

[13] Tosiyasu L. Kunii, "What's Wrong with Wrapper Approaches in Modeling Information System Integration and Interoperability?", Proceedings of the 3rd International Workshop on Databases in Networked Information Systems: User Interactions and Web Based Services, (DNIS 2003), September 22-24, 2003, The University of Aizu, Japan, Lecture Notes in Computer Science, Nadia Bianchi-Berthouze, Ed., pp. 86-96, Springer-Verlag, September, 2003.

[14] Tosiyasu L. Kunii, "Algebraic Topological Modeling for Cyberworld Design", Proceedings of International Conference on Cyberworlds, 3-5 December 2003, pp. xx-xxvi, Marina Mandarin Hotel, Singapore, IEEE Computer Society Press, Los Alamitos, California, U. S. A.

[15] Tosiyasu L. Kunii, "Homotopy Modeling as World Modeling", *Proceedings of Computer Graphics International '99 (CGI99),* (June 7-11, 1999, Canmore, Alberta, Canada) pp. 130-141, IEEE Computer Society Press, Los Alamitos, California, U. S. A.

[16] Paul Kennedy, "The Rise and Fall of the Great Powers," Random House, New York, 1987.

[17] T. L. Kunii, "Pax Japonica," (in Japanese), President Co., Ltd., Tokyo, October 1988.

[18] T. L. Kunii, "Creating a New World inside Computers - Methods and Implications-", Proc. of the Seventh Annual Conference of the Australian Society for Computers in Learning in Tertiary Education (ASCILITE 89), G. Bishop and J. Baker (eds.), pp. 28-51, Gold Coast, Australia, December 11-13, 1989, [also available as Technical Report 89-034, Dept. of Information Science, The University of Tokyo].

[19] T. L. Kunii and T. Noma, "Computer Graphics as a Tool to Visualize Virtual Worlds," Video Culture Canada's Second Annual International Festival "The New Media," November 2-6, 1984, Toronto.

[20] T. L. Kunii, "Electronic Alice's Wonderland," Graphics Interface 85, May 27-31, 1985, Montreal.

[21] Paul Chauchard, "Précis de Biologie Humaine – Les Bases Organiques du Compotement et de la Pensée -", Presses Universitaires de France, 1957.

[22] Julian S. Huxley, "Problems of Relative Growth, (Foundations of Natural History) ", Johns Hopkins University Press, 1993.

[23] J. S. Huxley, , G.. Teissier, "Terminology of relative growth", Nature, vol. 137, page 780, 1936a.

[24] Georges Teissier, "Allométrie de taille et variabilité", C. R. Soc. Biol., t. 124, p. 1071-1073, 1937.

[25] Tosiyasu L. Kunii, "Practicing Global Openness in Education: From Elementary Schools to Graduate Schools", dali2001(Digital and Academic Liberty of Information), March 26-29, 2001, Aizu-Wakamatsu, Japan.

[26] http://www.gnu.org/copyleft/gpl.html

[27] Richard M. Stallman, Lawrence Lessig (Introduction), Joshua Gay, "Free Software, Free Society: Selected Essays of Richard M. Stallman", Free Software Foundation; 2002.

[28] Tosiyasu L. Kunii, "The Potentials of Cyberworlds –An Axiomatic Approach-", Proceedings of International Conference on Cyberworlds, 18-20 November 2004, pp. 2-7, Tokyo, Japan, IEEE Computer Society Press, Los Alamitos, California, U. S. A.

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