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Cyber Graphics

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Abstract

Cyber graphics as an emerging technology has been playing key roles as human interfaces of cyber worlds. With the ever increasing roles of cyber worlds in the real world, as seen in e-financial trading that deals GDP-equivalent in a day, understanding of cyber graphics is becoming essential. To this end, scientific research has been conducted to grasp the foundation as invariants. Cellular modeling has been found to be of key importance. The adjunction spaces and cell attaching functions of cyber graphics are investigated through varieties of examples to find out the invariants successfully.

Keywords: adjunction spaces, cell attaching functions, a hierarchy of invariants, cellular spatial structures

1. What is "cyber graphics"?

Before going into the subject of cyber graphics, we need to clarify the meaning of *cyber worlds*. When we talk on cyber worlds, there are always discussed in connection with activities on the Web such as e-commerce, e-business, e-manufacturing and e-finance. Particularly, having e-financial trading reach to the amount GDP-equivalent, the roles of cyber worlds in the real world have been established. Thus, cyber worlds are quite different from virtual reality and also from augmented reality where virtuality is the emphasis.

Cyber graphics stands for computer graphics in cyber worlds. Computer graphics displays objects on computer screens for human cognition. Thus, computer graphics serves as a type of human interfaces to identify *cognitive objects* in human *cognitive spaces* with *cyber objects* in computational spaces on the Web called *cyber spaces*. In both spaces, objects change through cognitive processes and through computational processes. The identification is established through invariants through such changes. "Seeing is believing". Human interfaces heavily rely on visualizing objects by cyber graphics as shapes for display on the screens. Then what are shape invariants and what is an abstraction hierarchy of shape invariants? This issue is the essence of "science of cyber graphics" and also of "science of cyber worlds". Investigations on cognitive spaces and cognitive worlds are left as different researches.

2. An incrementally modular abstraction hierarchy of shape invariants

In terms of the abstraction of invariants hierarchically organized from general to specific to realize incrementally modular shape design of objects, the following is a reasonable case of an *incrementally modular abstraction hierarchy*:

1 An Extension Level, a Homotopy Level as a special case;
2 A Set Level;
3 A Topology Level, a Graph Theoretical Level as a special case;
4. A Cellular Structured Space Level;
5 A Geometry Level;
6 A Visualization Level.

For web graphics, "A Cellular Structured Space Level" based on cellular spatial structures [1, 2, 4] that include CW-complexes as a special case provides web graphics with a far more versatile theoretical foundation than that based on a graph theoretical level that is common in conceptual- and data- modeling [5,6], allowing cyber graphics to specify objects in cognitive- and computational- spaces as cells and their cellular attachment to form *adjunction spaces* (also called *attaching spaces*, *glued spaces*, and *adjoining spaces*). Cellular modeling also allows cellular composition and decomposition while maintaining *cell dimensions* and *cell connectivity* as invariants by preserving the homotopy.

Object identification is carried out systematically through an *identification mapping* (often called a *quotient mapping*) [3]. Here, *cell dimensions* mean the degrees of freedom, generally called *inductive dimensions*. For instance, on a point we have no degree of freedom to move and hence the dimension of a point is 0. On a line we can move on it from one point to another in any direction and hence the dimension of a line is 1. Likewise, the dimensions of a surface and a ball are 2 and 3. The *cell connectivity* is defined by a continuous mapping called a *cell attaching map*.

3. Adjunction spaces as shape invariants in geometrically changing situations and in geometrically indefinable situations

Cyber graphics relates various situations with the reality in the real world. As a simple example, Figure 1 shows the situation of wearing a hat. Our question is how to represent the situation by cyber graphics? The shape of the hat deforms after wearing at least a little bit. Also how deep the hat is worn varies on the occasions. Such situations and occasions make geometry inapplicable to represent the situation of Figure 1 by cyber graphics. This means that in the incrementally modular abstraction hierarchy, this situation has to be represented at above the geometry level.

Before wearing a hat.

After the hat is worn.

Figure 1. Wearing a hat.

Since the hat needs to be on and off, it cannot be joined to the head at the set theoretical level and hence the hat and the head have to be disjoint. This statement may sounds trivial, but actually when I submit papers that include this statement, I always get the reviews back saying that both are set theoretically joined. Then, how the hat is related to the head after the hat is worn? Figure 2 illustrates the relation by an attaching map f, and the situation "the cap is worn" as an adjunction space of two disjoint topological spaces X (the head) and Y (the cap), obtained by starting from the head X and by attaching the cap Y to the head via a continuous function f by identifying each point $y \in Y_0 | Y_0 \subseteq Y$ with its image $f(y) \in X$. Here Y_0 is the inside of the hat that touches the head, and the part of the head touched by the hat is $f(Y_0) \subseteq X$. After the hat is worn, thus,

$f(Y_0) \sim Y_0 | f(Y_0) \subseteq X, Y_0 \subseteq Y$.

Theorem 3.1 Given topological spaces X and Y, and an adjunction space $Y_f = X \sqcup_f Y = X \sqcup Y/\sim = X \sqcup Y / (x \sim f(y) \mid \forall y \in Y_0)$ obtained by attaching Y to X via a continuous function f by identifying each point $y \in Y_0 | Y_0 \subseteq Y$ with its image $f(y) \in X$, then $f(Y_0) \sim Y_0 | f(Y_0) \subseteq X$, $Y_0 \subseteq Y$.

Proof The proof is trivial. Y is assumed to be partitioned into a disjoint union of two equivalence classes such that $Y = Y_0 \sqcup (Y-Y_0)$. The attaching function f partitions X into a disjoint union of two equivalence classes such that $X = f(Y_0) \sqcup (X-f(Y_0))$. Hence, the reflexivity holds. The symmetry and transitivity is obvious from the continuity of the attaching function f. QED

After laying out the fundamental framework as explained above, we can see that the framework is quite general and applicable to versatile cases. To see this, let us look at another situation where geometrical representation cannot properly preserve invariants. It is a frill consisting of tucks [3, 7]. Usually a frill and tucks change shapes geometrically during the manufacturing processes and while being worn. Therefore, geometrical definition is not applicable. This means we have to find geometrical invariants to specify a frill and tucks. The geometrical invariants are being preserved and inherited while geometrical shape changes are taking place homotopically. What then are the geometrical invariants? This stands for a case of science of cyber graphics. The geometrical invariants are briefly sketched as follows and shown in Figure 3. A case of real tucked objects, coat sleeves, is shown in Figure 4. Each tuck consists of a particular type of fold, actually a pair of folds. The tuck is then attached to the other part through a cell attaching (also called adjoining or adjunction) operation. Before the cell attachment, the tuck is decomposed into three 1-cells and one 2-cell. The cell attachment here is a surjective and continuous function that maps a line on the collar to the three 1-cells of the tuck. The three 1-cells are topologically equivalent, and form an equivalence class.

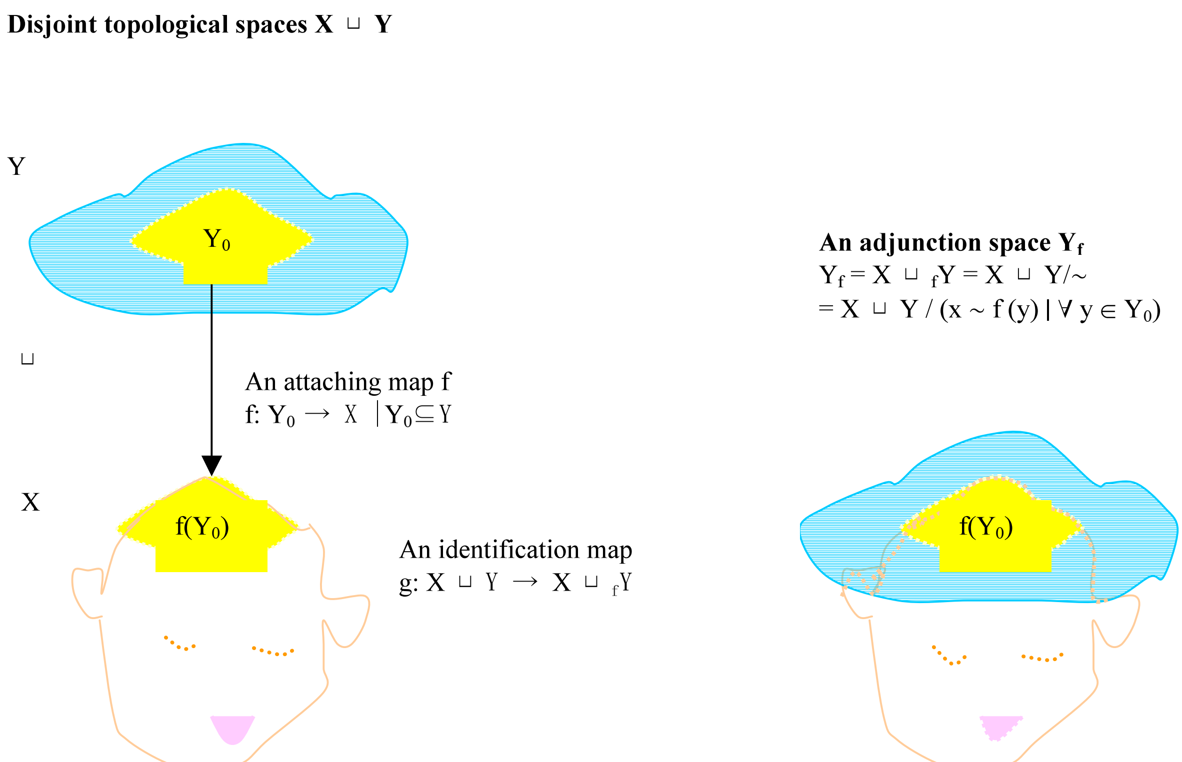

Figure 2. An adjunction space of two disjoint topological spaces X and Y, obtained by starting from X and by attaching Y to it via a continuous function f by identifying each point y ∈ Y_0 | $Y_0 \subseteq$ Y with its image f(y) ∈ X.

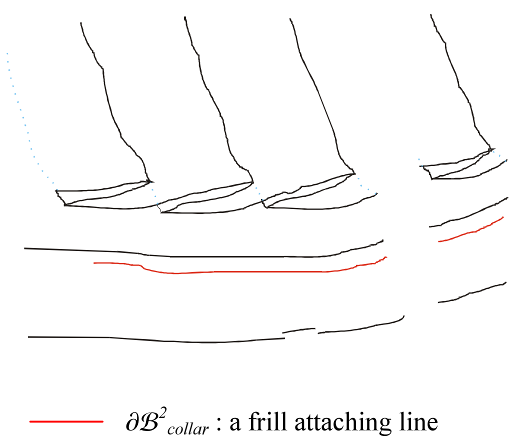

Fig. 3 Frill composition = Collar attachment to tucks via the attaching map $f\colon \partial\mathcal{B}^2_{collar} \to \sqcup_i (\mathcal{B}^1{}_i \sqcup \mathcal{B}^1{}_i \sqcup \mathcal{B}^1{}_1)_{tuck\ edge\ i}$.

Fig. 4 Coat sleeve tucks.

A further example is attaching a button as in Figure 5. As in Figure 6, we define button attachment similarly by two attaching functions and the result as two adjunction spaces. A button $\mathcal{B}^2_{\text{button}}$ as a 2-cell is attached to a garment $\mathcal{B}^2_{garment}$, a 2- cell, by a thread $\mathcal{B}^1_{thread}$, a 1-cell. First, an endpoint of the thread is attached to the button $\mathcal{B}^2_{\text{button}}$ via an attaching map:

$f: \quad \partial\partial\mathcal{B}^2_{\text{button}} \rightarrow \partial\mathcal{B}^1_{\text{thread}}$

to get an adjunction space

$\mathcal{B}^2_{button} \sqcup_f \mathcal{B}^1_{thread} /\sim$

.

The other end of the thread is attached to a specified point of the garment via

$g: \quad \partial\partial\mathcal{B}^2_{garment} \rightarrow \partial\mathcal{B}^1_{thread}$

to obtain an adjunction space

$\mathcal{B}^2_{garment} \sqcup_g \mathcal{B}^1_{thread} /\sim$.

Figure 5. A button attached to a garment.

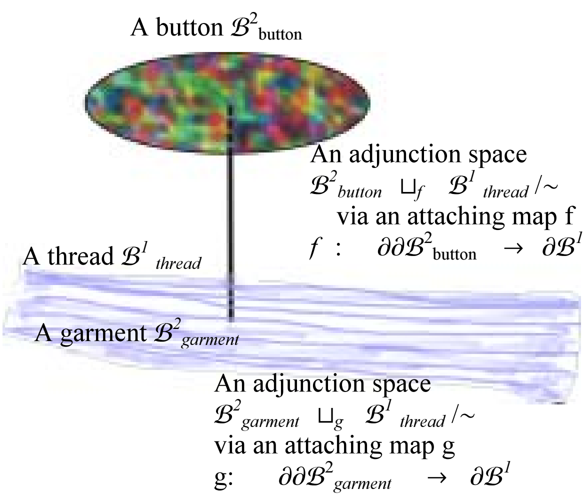

Figure 6. Adjunction spaces of button attachment.

4. Glossary of adjunction space photos

For your interest, a collection of photos is shown here so that you enjoy formulating adjunctions spaces by finding attaching functions.

5. Consequences of science of cyber graphics

Cyber graphics is an emerging key area as critical human interfaces in the Web era where cyber worlds are playing major roles in the real world as seen in e-financial trading that trades a GDP-equivalent amount a day. Still it is in its infancy as an academic discipline. Here we look at it from a scientific viewpoint. The meaning of science is its validity in modeling varieties of problems systematically. Therefore, we researched on invariants of cyber graphics here. In real world applications, the scientific findings recorded have big impacts. For example, industrial products such as copiers and fax machines process paper sheets that change geometry, making geometrical shape modeling invalid while cellular modeling is valid. In business graphics, we have to display business structures such as corporate M&A structures, business relationships, and electronic financial trading. They are time variants but still their cellular structures are invariants. Science of cyber graphics is here to be advanced in knowledge.

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