

Modeling of Conceptual Multiresolution Analysis by an Incrementally Modular Abstraction Hierarchy

Tosiyasu L. KUNII[†], *Member*, Masumi IBUSUKI^{†††}, *Nonmember*, Galina I. PASKO[†], *Nonmember*, Alexander A. PASKO^{††}, *Nonmember*, Daisuke TERASAKI^{††}, *Member* and Hiroshi HANAIZUMI^{††}, *Member*

Summary

Recent advances of Web information systems such as e-commerce and e-learning have created very large but hidden demands on conceptual multiresolution analysis for more generalized information analysis, cognition and modeling. To meet the demands in a general way, its modeling is formulated based on modern algebraic topology. To be specific, the modeling formulation is worked out in an incrementally modular abstraction hierarchy with emphasis on the two levels of the hierarchy appropriate for conceptual modeling: the adjunction space level and the cellular structured space level. Examples are shown to demonstrate the usefulness of the presented model as well as an implementation of a flower structure case.

Key words:

conceptual multiresolution analysis, adjunction spaces, cellular structured spaces, incrementally modular abstraction hierarchy, Web information systems

1. Introduction

Traditionally, researches on multiresolution image analysis have been on hierarchically organized multiresolution images, usually represented as image pyramids [1] composed of hierarchies of images, for example, triangle-based [2] and voxel-based [3] hierarchies.

There have been very large but hidden demands on the modeling of *conceptual multiresolution analysis* in general. They have been hidden because of the unavailability of the models to explicitly meet the needs of analyzing rapidly developing Web information systems to cognize and model varieties of information on the Web. Applications include e-commerce to cognized necessary merchandises at shops on the Web, and e-learning as a type of distance education to quickly learn necessary knowledge on the Web. Our model is based on an incrementally modular abstraction hierarchy, each level of which corresponds to the abstractness of concept. It is in a big contrast with the traditional multiresolution analysis that classifies images as a hierarchy of geometrical objects, either physical or logical. Conceptual multiresolution is a case of more general information multiresolution levels where objects are expressed by using general information such as their features and properties.

The traditional multiresolution analysis has been also centered on the *static* hierarchical relations of the objects. For example, the relations between the receptacle of a flower and its petals present a static hierarchical expression of the flower. However, *dynamic relations* such as ‘petals scatter after full bloom’ are hardly represented. Usually physical behaviors of dynamics systems are formulated by a set of time dependent partial differential equations. It is unrealistic in our cases because of the complexity of the objects we are considering for multiresolution analysis. Particularly, to represent the dynamism conceptually, the relationships between dynamic objects bear meanings that are more abstract. To carry out the multiresolution analysis automatically as much as possible to save troublesome manual work, let us model conceptual multiresolution analysis in formal ways.

2. The Framework of Modeling

We consider conceptual multiresolution analysis based on the following levels of an abstraction hierarchy that is incrementally modular [4, 5]:

1. An Extension Theory Level, a Homotopy Theoretical Level as a special case.
2. A Set Theoretical Level.
3. A Topology Theoretical Level, a Graph Theoretical Level as a special case.
4. An Adjunction Space Level.
5. A Cellular Structured Space Level.
6. A Geometry Level.
7. A View Level

At the level 3, the topology we adopt for modeling is *discrete topology*. The levels 6 and lower are application domain dependent, and they are defined here for the multiresolution analysis domain. In this paper, we elaborate on the adjunction space level 4 and the cellular structured space level 5 to model conceptual multiresolution analysis in necessary details.

At the adjunction space level, a dynamic relation of the type we explained above is expressed by an equivalence relation f (symbolically denoted by \sim) between objects such that an object X has become related via f with another object Y by sharing a part $Y_0 \subseteq Y$ such that $x \sim f(y) \mid \exists x \in X, \forall y \in Y_0$. There are numerous cases of the adjunction

[†]IT Institute, [†]Kanazawa Institute of Technology, 1-15-13 Jingu-mae, Shibuya-ku, Tokyo 150-0001 Japan.

^{††}Department of Digital Media Science, Graduate School of Computer and Information Sciences, Hosei University.

3-7-2 Kajino-cho, Koganei City, Tokyo, Japan 184-8584.

^{†††}Dame Dental Clinic, 2-27-25 Minami Aoyama, Minato-ku, Tokyo, Japan 107-0062.

spaces, for example, those when industrial products are assembled and when any trading is conducted.

At the cellular structured space level, the *inductive dimension* n of each object defined at the adjunction space level is added as the degrees of freedom of the object. We abbreviate an inductive dimension to a *dimension* in this paper for simplicity. For example, usually the shapes of industrial products are 3 dimensional, and the commodities traded have numbers of features as their dimensions.

The geometry level corresponds to the geometrical multiresolution. At this level, geometrical properties such as coordinate systems and metric systems are added to the objects at the cellular structured space level. Since there are abundant researches on multiresolution analysis at this level [1], we do not elaborate the geometrical level any further.

Before going through different cases of conceptual multiresolution analysis at the adjunction space level and the cellular structured space level based on the incrementally modular abstraction hierarchy, let us briefly look at the higher levels of the incrementally modular abstraction hierarchy. To do so, let us use an example of conceptual multiresolution analysis such that it is very far from the traditional geometrical multiresolution analysis. It is a case of conceptual multiresolution analysis of e-commerce of books that does not have a geometrical level. For any objects to be computable by computers that are set theoretical automatic machines, they have to be defined in set theoretical spaces. We say objects are at *the set theoretical space level*.

There is one level higher abstract level that is the homotopy theoretical level that is a typical case of extension theoretical levels. At the *homotopy theoretical level*, homotopy can be preserved by making any changes reversible. By defining reversible operations to add elements to a given set or delete them from it, we can create a level one level higher than the set theoretical level. That is the homotopy theoretical level, and is more abstract than the set theoretical level.

Any objects we look at usually have subsets as their elements, and hence, by definition, they are *discrete topological spaces*. For example, in the following case of online book shopping, a customer X has the given name, the middle name and the surname as the ‘name’, making a subset {the surname, the middle name, the given name} as an element ‘name’ of X . Any set theoretical space with its subsets as elements is basically a discrete topological space. Likewise, since an online bookstore Y has subsets, new books and used books, as its elements, Y is a discrete topological space too. Hence, X and Y are at the *discrete topology theoretical level*. Hereafter, we omit ‘discrete’, and denotes discrete topology simply as topology.

3. Conceptual Multiresolution Analysis of e-Commerce: A Case of Online Book Shopping

3.1 The Adjunction Space Level

Eventually a case of online book shopping is analyzed at the level one level lower than the topological theory level as the analysis of adjunction spaces. Let us suppose that a customer X is surfing on the Web and has found an online bookstore Y . We assume as explained before that X and Y are topological spaces.

Initially, the customer X and the online bookstore Y are assumed independent of each other, and hence the initial state is denoted as $X \sqcup Y$ where \sqcup stands for disjoint union (also called exclusive disjunction). Sometimes $+$ is used in place of \sqcup .

Let us suppose that the customer X has found the books Y_0 posted on the Web at the home page of an online bookstore Y during Web surfing as we do window-shopping for goodies. It is a Web window-shopping process. Since the books Y_0 are a part of the properties of the online bookstore Y , $Y_0 \subseteq Y$ holds. The processes of online book shopping on the web as e-commerce are analyzed and represented as shown in Fig. 1. Fig. 1 illustrates how the customer X becomes related to the online bookstore Y after the books Y_0 are identified for trading.

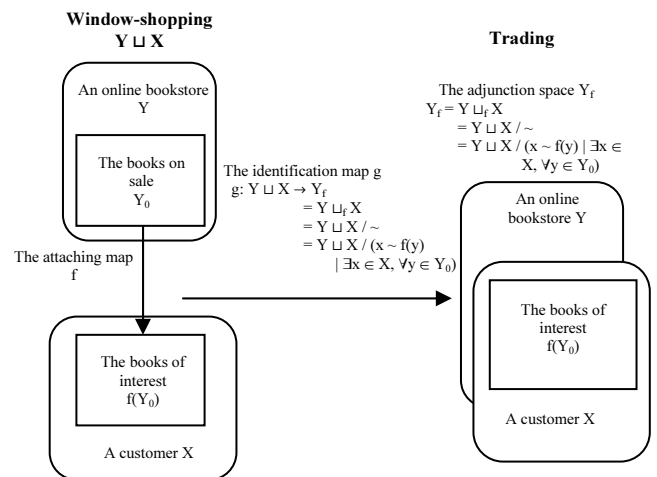


Fig. 1 Conceptual multiresolution analysis of an online bookstore at the adjunction space level.

The *identification map* (also called the *quotient map*) plays a central role to show how the original situation such that there are a customer X and an online bookstore Y , namely $Y \sqcup X$, is related to the situation after the customer X identified the books of interest $f(Y_0)$ such that X and Y form an adjunction space $Y_f = Y \sqcup_f X = Y \sqcup X / \sim = Y \sqcup X / (x \sim f(y) \mid \exists x \in X, \forall y \in Y_0)$ relating the books Y_0 of the online bookstore Y with the books of interest $f(Y_0)$ to the customer X .

This is a case of *dynamic relations* we explained before. The adjunction space level we present here precisely represents the relation by an attaching map f , and also represents the situation “the books are identified for trading” as the adjunction space of two disjoint topological

spaces X (the customer) and Y (the online bookstore), obtained by starting from the customer X and by attaching the online bookstore Y to the customer X via a continuous map f by *identifying* each point $y \in Y_0 \mid Y_0 \subseteq Y$ with its image $f(y) \in X$ so that $x \sim f(y) \mid \exists x \in X, \forall y \in Y_0$. Thus, the equivalence denoted by a symbol \sim plays the central role to compose adjunction spaces at the adjunction space level of the conceptual multiresolution analysis.

Thus, the adjunction space of online book shopping at a online bookstore Y by a customer X on the books Y_0 is formulated along the line explained so far as follows.

The *adjunction space* Y_f (also called the *attaching space*, the *adjoining space*, and the *pasted space*)

$$Y_f = Y \sqcup_f X = Y \sqcup X / \sim = Y \sqcup X / (x \sim f(y) \mid \exists x \in X, \forall y \in Y_0)$$

is obtained by *identifying* each point $y \in Y_0 \mid Y_0 \subseteq Y$ with its image $f(y) \in X$ so that $x \sim f(y) \mid \forall y \in Y_0$. This represents the customer X 's book shopping process as a dynamic situation. To be more precise in explaining the dynamic situation, from a set of books Y_0 at the online bookstore Y , the customer X select a book of interest $f(y)$ by making it an element of the customer X 's interest in book shopping that is naturally an element x of the customer X . Hence, $x \sim f(y)$. The customer X continues shopping to this way to identify numbers of books of interest. The attaching map f and the identification map g are:

$$\begin{aligned} f: Y_0 &\rightarrow X \mid Y_0 \subseteq Y, \text{ and} \\ g: Y \sqcup X &\rightarrow Y_f = Y \sqcup_f X = Y \sqcup X / \sim = Y \sqcup X / (x \sim f(y) \mid \exists x \in X, \forall y \in Y_0). \end{aligned}$$

The *identification map* (also called the *quotient map*) shows how the original situation such that there are a customer X and an online bookstore Y , namely $Y \sqcup X$, is related to the situation after the customer X identified the books of interest $f(Y_0)$ such that X and Y form an adjunction space $Y_f = Y \sqcup_f X = Y \sqcup X / \sim = Y \sqcup X / (x \sim f(y) \mid \exists x \in X, \forall y \in Y_0)$ relating the books Y_0 of the online bookstore Y with the books of interest $f(Y_0)$ to the customer X .

3.2 The Cellular Structured Space Level

Let us analyze the online book shopping at the one lower level of the incrementally modular abstraction hierarchy for the conceptual multiresolution analysis. Analyzing and finding the dimensions of topological spaces at the adjunction space level create this level. Suppose we have found that the dimension of the customer X is n , that of the online bookstore is m , and that of the books Y_0 at the online bookstore is k ($k \leq m$). Let us denote a closed n dimensional topological space called a *closed n -cell* \mathcal{B}^n . Then, the online bookstore is denoted as $\mathcal{B}^m_{\text{store}}$, the customer as $\mathcal{B}^n_{\text{customer}}$, and the books $\mathcal{B}^k_{\text{store}}$. The adjunction space is:

$$\mathcal{B}^m_{\text{store}} \sqcup_f \mathcal{B}^n_{\text{customer}} = \mathcal{B}^m_{\text{store}} \sqcup \mathcal{B}^n_{\text{customer}} / \sim = \mathcal{B}^m_{\text{store}} \sqcup \mathcal{B}^n_{\text{customer}} / (x \sim f(y) \mid \exists x \in \mathcal{B}^n_{\text{customer}}, \forall y \in \mathcal{B}^k_{\text{store}}). \text{ The attaching map } f \text{ and identification map } g \text{ are:}$$

$$\begin{aligned} f: \mathcal{B}^k_{\text{store}} &\rightarrow \mathcal{B}^n_{\text{customer}} \quad (\text{where, } \mathcal{B}^k_{\text{store}} \subseteq \mathcal{B}^m_{\text{store}}) \\ g: \mathcal{B}^m_{\text{store}} \sqcup \mathcal{B}^n_{\text{customer}} &\rightarrow \mathcal{B}^m_{\text{store}} \sqcup_f \mathcal{B}^n_{\text{customer}} \end{aligned}$$

And the books at the online bookstore are

$$\partial^{m-k} \mathcal{B}^m_{\text{store}} = \mathcal{B}^k_{\text{store}}.$$

3.3 The Geometry Level

The buying and selling of the book is not geometrical, and hence the conceptual multiresolution analysis of online book shopping does not produce the geometry level.

4. Conceptual Multiresolution Analysis of Flowers

From now on, let us analyze the conceptual multiresolution levels of e-learning through two subjects: botany and dentistry as popular themes. In botany, we look at flowers as a popular theme.

4.1 The Adjunction Space Level

On the Web there are flowers with clear illustrations of structures. Fig. 2 and Fig. 3 are excellent examples [6, 7]. In e-learning on the Web, we learn the stages of life of flowers. They become in full bloom and after a while they are gone, leaving the petals scattered on the ground beautifully and serving as a favorite theme of poetry. In modeling conceptual multiresolution analysis, this means given any flower, for the petals to fall, they cannot be joined to the receptacle of the flower at the set theoretical level. This further means the petals and the receptacle have to be disjoint. Then, how are the petals related to the receptacle before the petals are fallen? This relation reflects the situation "the petals are attached to the receptacle such that they fall after a while". The adjunction space represents the situation as it is.

Let us denote the receptacle of a flower as a topological spaces X and the i -th petal ($i = 0, 1, 2, 3, \dots$) as a topological space Y_i . The adjunction space is a topological space obtained by starting from the receptacle X and by attaching the petals $\sqcup_i Y_i$ to the receptacle via a continuous map f by identifying each point $x \in X \mid Y_{0i} \subseteq Y_i$ with its image $f(y_i) \in X$ for all $y_i \in Y_{0i}$ and for $i = 0, 1, 2, 3, \dots$. Here Y_{0i} is the edge of the i -th petal that touches the receptacle, and the part of the receptacle X touched by the i -th petal is $f(Y_{0i}) \subseteq X$. Before the petals are fallen, thus, the following equivalence relation holds:

$$x \sim f(y_i) \mid \exists x \in X, \forall y_i \in Y_{0i}, Y_{0i} \subseteq Y_i \text{ for } i = 0, 1, 2, 3, \dots$$

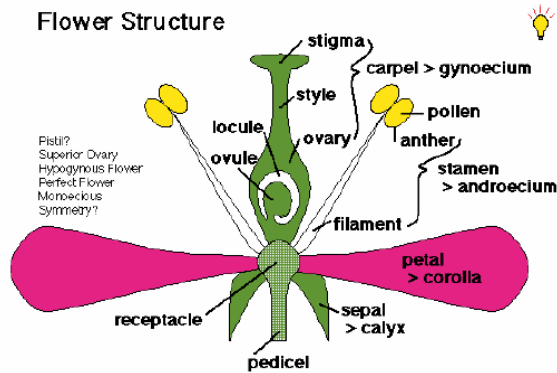


Fig. 2 Flower structure
http://koning.ecsu.ctstateu.edu/plants_human/flowerstructure.html [6].

<http://www.mendelweb.org/MWflower.html>

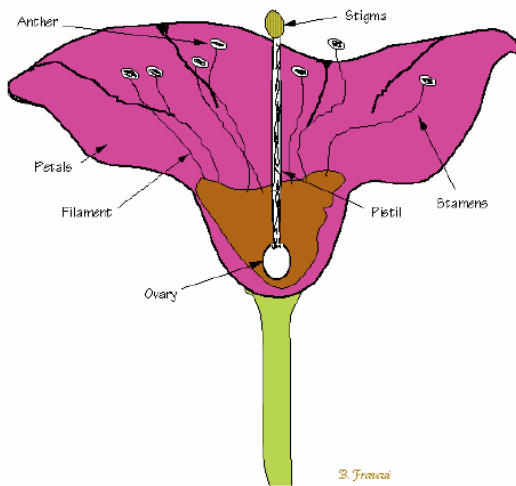


Fig. 3 Flower structure illustrated as the reproduction part of the plant [7].

Hence, the adjunction space Y_f consisting of the petals and the receptacle is:

$$Y_f = \sqcup_i Y_i \sqcup_f X = \sqcup_i Y_i \sqcup X / \sim = \sqcup_i Y_i \sqcup X / (x \sim f(y_i) \mid \exists x \in X, \forall y_i \in Y_{0i} \mid i = 0, 1, 2, 3, \dots)$$

The identification map g relates

- ★ the situation A) such that $\sqcup_i Y_i \sqcup X$, that is, there are a set of petals and the receptacle, independent of each other

with

- ★ the situation B) such that $\sqcup_i Y_i \sqcup_f X$, that is, a set of petals form a part of the flower such that the set of petals are attached to the receptacle at its part where the petals touch the receptacle.

To be more precise, the bottom parts of the petals are attached to the top part of the receptacle. The part of the petals $\sqcup_i Y_{0i} \subseteq \sqcup_i Y_i$ is touched to the part $f(\sqcup_i Y_{0i}) \subseteq X$ of the receptacle X , and both parts become shared to form equivalence classes. The identification map g is $g: \sqcup_i Y_i \sqcup_f X \rightarrow Y_f = \sqcup_i Y_i \sqcup_f X$. The situation B is, for example, the flower in bloom, and A is, for example, the blossom is gone.

4.2 The Cellular Structured Space Level

The cellular structured space of flower is obtained by endowing dimensions to the topological spaces at the adjunction space level. This is one level lower than the adjunction space level in conceptual multiresolution analysis. This process to generate the cellular structured space level is almost trivial and proceeds as follows.

The petals are closed 3 dimensional cells $\sqcup_i Y_i = \sqcup_i \mathcal{B}^3_{\text{petal}, i}$. The receptacle $X = \mathcal{B}^3_{\text{receptacle}}$ of the flower is a closed 3 dimensional cell. The surfaces $\sqcup_i Y_{0i} = \sqcup_i \mathcal{B}^2_{\text{petal}, i} \subset \sqcup_i Y_i = \sqcup_i \mathcal{B}^3_{\text{petal}, i}$ that are the parts of petals attached to the top part surface of the receptacle $f(\sqcup_i Y_{0i}) = f(\sqcup_i \mathcal{B}^2_{\text{petal}, i}) \subset X = \mathcal{B}^3_{\text{receptacle}}$.

The adjoining space at the cellular space level is, then:

$$\sqcup_i Y_i \sqcup_f X = (\sqcup_i \mathcal{B}^3_{\text{petal}, i}) \sqcup_f \mathcal{B}^3_{\text{receptacle}} = (\sqcup_i \mathcal{B}^3_{\text{petal}, i}) \sqcup \mathcal{B}^3_{\text{receptacle}} / \sim = (\sqcup_i \mathcal{B}^3_{\text{petal}, i}) \sqcup \mathcal{B}^3_{\text{receptacle}} / (x \sim f(y) \mid \exists x \in \mathcal{B}^3_{\text{receptacle}}, \forall y \in \sqcup_i \mathcal{B}^2_{\text{petal}, i} \mid i = 0, 1, 2, 3, \dots)$$

Here, the attaching map f and the identification map g are at the cellular space level of conceptual multiresolution analysis of the flower:

$$f: \sqcup_i \mathcal{B}^2_{\text{petal}, i} \rightarrow \mathcal{B}^3_{\text{receptacle}} \mid \mathcal{B}^2_{\text{petal}, i} \subset \mathcal{B}^3_{\text{petal}, i}$$

$$g: (\sqcup_i \mathcal{B}^3_{\text{petal}, i}) \sqcup \mathcal{B}^3_{\text{receptacle}} \rightarrow (\sqcup_i \mathcal{B}^3_{\text{petal}, i}) \sqcup_f \mathcal{B}^3_{\text{receptacle}}$$

And, the surfaces of the petals attached to the receptacle are:

$$\sqcup_i \partial \mathcal{B}^3_{\text{petal}, i} = \sqcup_i \mathcal{B}^2_{\text{petal}, i}$$

$$\partial \mathcal{B}^3_{\text{receptacle}} = \mathcal{B}^2_{\text{receptacle}}$$

4.3 The Geometry Level

Usually flower structures are illustrated as 2 dimensional (2D) annotated drawings as typically seen in Fig. 2 and 3 [6, 7]. They can be classified geometrically as 2D shapes of the layered structure in conceptual multiresolution analysis. They serve to present the conceptual structures. By reformulating them in our framework, particularly at the adjunction space level and the cellular space level, conceptually we can more clear relationships of the named elements of flowers, such as whether some elements are touching each other at some surfaces or they are parts of the same larger element, by attaching functions and

identification functions. If they are touching each other, the disjoint unions \sqcup hold, and if they are parts of a larger element, set theoretical unions \cup hold. Then, we can reformulate usual botanical explanations that go, for example, as follows: ‘The flower consists of the part of the androecium, the pistil, and the plinth, the androecium consists of the part of the anther and the filament; the pistil consists of the part of stigma, the style, and the ovary, and the plinth consists of the part of petals, the receptacle, and the calyx’. To carry out the reformulation, we need to go back to the real flowers that are 3D for reanalysis. We are currently working on flowers of anemone and cosmos (Fig. 4). The upper part of Fig. 4 shows the flower of anemone.



Fig. 4 Anemone and cosmos flowers.

The lower part shows the entire flower (the left side) and a part of the pistil and the androecium (the right side) of cosmos. The reanalysis requires extensive work on the nature of the attachments of the elements at various levels of details including the biological cell levels and molecular biology levels. It provides us with interesting research themes.

5. Conceptual Multiresolution Analysis of Dental Modeling

Suppose we wish to take a dentistry course on the Web at a distance education site. There are many outstanding courses offered. Materials posted at the course Web sites are quite solid and well established, making them dependable. It will be helpful for such courses to use the conceptual multiresolution analysis we present here to make them more comprehensive so that e-learning and e-

education become more interactive to give the levels of details when they become needed at each stage of distance education via e-education.

Let us examine typical dentistry subjects. Fig. 5 is the pantomography of the teeth of one of the author (TLK). The FDI (International Dental Federation) / ISO two digit dental numbering system (also called the *FDI notation*) is shown in Fig. 6 [8]. In Fig. 5, the teeth 18, 38, and 48 are unerupted, and the tooth 28 is missing because it was extracted.

Usually, teeth are mutually disjoint. Let us denote the tooth labeled by n in the FDI notation as T_n . In the case the artificial crown extends over two or more teeth, how such teeth are related with each other? In Fig. 7, the X-ray photograph shows the teeth from 13 to 17 in the resolution physically higher than that of the pantomography. The crown extends over the two teeth, T_{13} and T_{14} , making them joined to form $T_{13} \cup T_{14}$, not $T_{13} \sqcup T_{14}$ any longer.

How the teeth are related to the gums? Since the teeth can be pulled out, they cannot be joined either to the gingivae (the gums) or to the alveolar bone at the set theoretical level. Hence, the teeth have to be disjoint with the gingivae and the alveolar bone as properly represented at the adjunction space level. Such information regarding the conceptual multiresolution analysis is at the adjunction space level, and complements the information on the structure of a normal tooth shown in Fig. 8. [9] and Fig. 9 [10].

The tissues that hold the tooth to the jawbone are known as the *periodontium*. It is made of the following tissues: the gingiva (the gum), the periodontal ligament, the root cementum of the tooth, and the alveolar and supporting bone. As shown in Fig. 10, the diagrammatic view of the periodontium structures (adjacent to the cervical region of a mandibular tooth) is very complex [11].

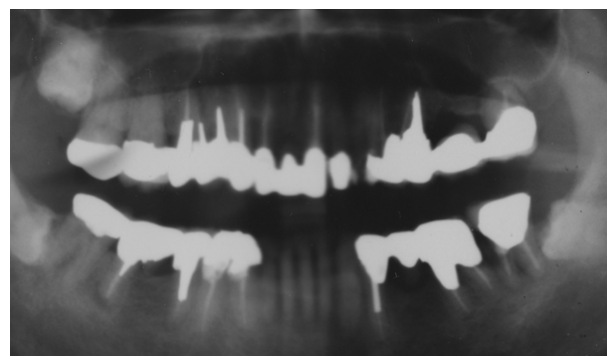


Fig. 5 The pantomography of the teeth of one of the author (TLK). The tooth 28 is extracted. The teeth 18, 38 and 48 are unerupted.

	18	17	16	15	14	13	12	11		21	22	23	24	25	26	27	28	
R	48	47	46	45	44	43	42	41		31	32	33	34	35	36	37	38	L

Fig. 6 The FDI (International Dental Federation) / ISO two digit dental numbering system (also called the *FDI notation*) where R stands for “right” and L for “left”.

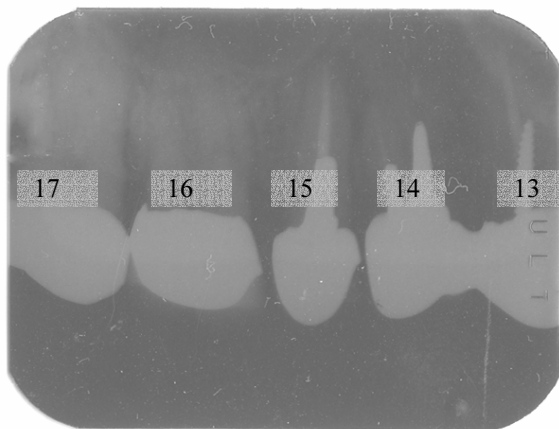


Fig. 7 The X-ray photograph of the teeth from 13 to 17.

http://animaldiversity.ummz.umich.edu/anat/tooth_structure.html [10]

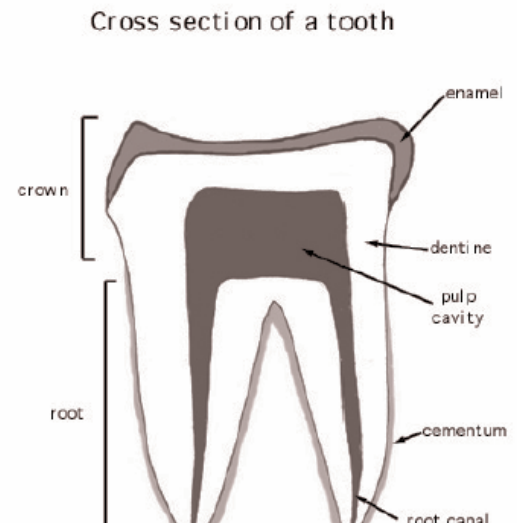
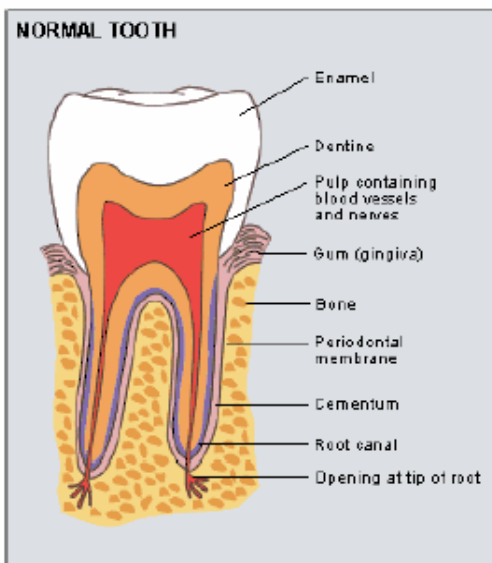


Fig. 9 Cross section of a tooth [10].

TOOTH: STRUCTURE OF A NORMAL TOOTH



<http://www.mydr.com.au/default.asp?article=3728> [9].

Fig. 8 Structure of a normal tooth [9].

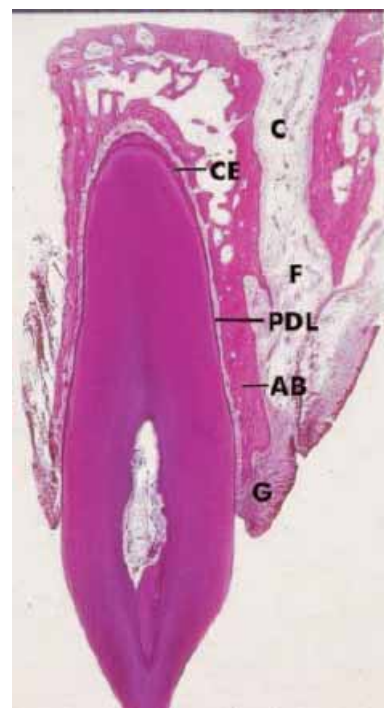


Fig. 10 The periodontium as shown in [11]: Histologic section of a maxillary central incisor.

Note that the enamel is missing as a result of demineralization during histologic processing. The cementum layer (CE) is connected to the adjacent alveolar bone (AB) by a thin periodontal ligament (PDL). The gingiva covers the bone and attaches to the cervical region of the tooth. The incisive canal (C) and incisive foramen (F) can be seen distinctly on the palatal aspect of the tooth. (G), palatal gingiva.

In this paper, as the initial work on modeling the dental conceptual multiresolution analysis, we research on the relationships of the teeth and periodontium keeping the analysis and modeling on the detailed structure of the periodontium as the next subject. It is a challenging subject bringing in very interesting problems on dental sciences.

5.1 The Adjunction Space Level

Fig. 8 illustrates the structure of a normal tooth [9]. Fig. 9 shows the cross section of a tooth [10]. They are all posted on well-developed Web sites for e-education. What is illustrated in Fig. 8 is a general relation as the common situation of a normal tooth. In our modeling of conceptual multiresolution analysis, what is meant at the global level of details on the relationship between the tooth and the periodontium without going into the internal structure of them? It is quite simple and is the situation such that “the tooth is invested and supported by the periodontium” as an adjunction space of a topological spaces X (the periodontium) and Y_i (i.e. the topological space to represent the i -th tooth T_i), obtained by starting from the periodontium X and by attaching the tooth Y_i to the periodontium via a continuous map f by identifying each point $y_i \in Y_{0i} \mid Y_{0i} \subseteq Y_i$ with its image $f(y_i) \in X$ to form an adjunction space $Y_f = Y_i \sqcup_f X$. Here Y_{0i} is the part of the surface of the root of the i -th tooth that touches the periodontium, and the part of the surface of the periodontium touched by the i -th tooth is $f(Y_{0i}) \subseteq X$. Thus, the attaching function f is:

$$f: Y_{0i} \rightarrow X \mid Y_{0i} \subseteq Y_i$$

The identification function g is:

$$\begin{aligned} g: Y_i \sqcup X &\rightarrow Y_f \\ &= Y_i \sqcup_f X = Y_i \sqcup X / \sim \\ &= Y_i \sqcup X / (x \sim f(y_i) \mid \exists x \in X, \forall y_i \in Y_{0i}). \end{aligned}$$

The X-ray image in Fig. 7 presents two typical cases of the relationships between the teeth.

The first is a relationship between the two normal teeth. The teeth T_{16} and T_{17} are touching with each other, and is modeled as an adjunction space of disjoint topological spaces Y_{16} (the topological space to represent the 16th tooth T_{16}) and X_{17} (the topological space to represent the 17th tooth T_{17}), obtained by starting from X_{17} and by attaching the Y_{16} to X_{17} via a continuous map f by identifying each point $x_{0,17} \in X_{0,17} \mid X_{0,17} \subseteq X_{17}$ with its image $f(y_{0,16}) \in X_{17}$. Here $Y_{0,16}$ is the topological space to represent the part of the side of the 16th tooth T_{16} that touches the 17th tooth T_{17} , and $X_{0,17}$ is the part of the side of the 17th tooth T_{17} touched by the 16th teeth $f(Y_{0,16}) \subseteq X_{17}$. The topological spaces Y_{16} and X_{17} that represent the teeth T_{16} and T_{17} respectively thus form an adjunction space $Y_{16} \sqcup_f X_{17}$. The attaching map f is then:

$$f: Y_{0,16} \rightarrow X_{17} \mid Y_{0,16} \subseteq Y_{16}$$

The identification map g is:

$$\begin{aligned} g: Y_{16} \sqcup X_{17} &\rightarrow Y_{16} \sqcup_f X_{17} = Y_{16} \sqcup X_{17} / \sim \\ &= Y_{16} \sqcup X_{17} / (x_{0,17} \sim f(y_{0,16}) \mid \exists x_{0,17} \in X_{17}, \\ &\quad \forall y_{0,16} \in Y_{0,16}). \end{aligned}$$

The second typical case is, as explained before, the case two teeth are related artificially by a shared crown. The teeth T_{13} and T_{14} are joined having the crown extending over them. They are not disjoint but joined at the set theoretical level, namely $Y_{13} \cup X_{14}$ where Y_{13} and X_{14} are topological spaces to represent teeth T_{13} and T_{14} respectively.

Fig. 11 is the photo of a plaster cast of the teeth. When the teeth are closed together, the upper teeth are matched to the lower teeth. The upper teeth and the lower teeth have to be disjoint so than they can be touched or detached freely. The attaching function and the identification function are basically similar to those of the cases explained so far.



Fig. 11 A plaster cast of the teeth.

There are different aspects. The canines are for cutting food and the molars for crushing food. Our previous research has revealed that the canines and the molars exhibit quite different dynamic attaching patterns while masticating food. The research on this in 1995 was based on *differential topology*, the Morse theory and the Reeb graph in particular [12, 13], and it bridges between the algebraic topological researches presented in this paper and the geometrical researches on dentistry.

5.2 The Cellular Structured Space Level

We can almost automatically derive all the results of conceptual multiresolution dental analysis at the cellular structured space level from those at the adjunction space level presented above, by endowing any topological space with its dimension n to turn it into a closed n -cell \mathcal{B}^n . If the dimension is 2, the topological space is turned into \mathcal{B}^2 and if 3 into \mathcal{B}^3 .

5.3 The Geometry Level

Though both Fig. 5 and Fig. 7 are types of X-ray images of the teeth and the periodontium, as explained above, the resolution of Fig. 7 is physically higher than that of Fig. 5. This type of physical resolution difference is at the geometry level and belongs to the category of the geometrical multiresolution analysis. Conceptually, the real problems of multiresolution analysis at the geometrical level lie in the coexistence of orders-of-magnitude different precisions. The teeth move usually in the orders of a few centimeters while masticating food and their occlusion often requires the precision of a few microns. The ratio is $1:10^{-6}$. We have conducted extensive researches on the problems. The researches have presented very difficult problems of geometrically controlling the high precision tooth occlusion by the loose coupled jaw joint mechanism [14 -17]. From the viewpoint of modeling conceptual multiresolution analysis based on the incrementally modular abstraction hierarchy, we need to conduct further researches on such geometrical modeling results from the side of the algebraic topological modeling proposed in this paper. So far the differential topological aspects [12 - 13] are mainly related. There are many excellent general references on algebraic topology on the Web such as those by Allen Hatcher [18].

6. Implementation: A Case of Conceptual Multiresolution Analysis of Flowers

The implementation of the geometrical level is conducted to further demonstrate how we carry out conceptual multiresolution analysis for representing the type of the dynamism “the petals are attached to the receptacle such that they fall after a while” as mentioned in 4 so that we can display it as images. The implementations at the adjunction space level and the cellular structure space level are already given in 4 in detail. Not all geometrical representations are incrementally modular. The nature of the types of the geometrical levels that are incrementally modular is a quite interesting and fundamental research theme.

As one of such geometrical levels, we present a function representation (FRep) [19] that can directly inherit cellular spatial structures such as dimensions, cell boundaries, and cell attachments. The images created based on the geometrical implementation are illustrated in Figs.12-14. In Fig. 12, the flower pedicel with the receptacle and five attached petals are shown. The function representation (FRep) and its supporting language (www.hyperfun.org) are used to model and render 3D solid objects using lower dimensional cells. The pedicel and the receptacle are modeled as convolution surfaces with a curve and a point as corresponding skeletons. Each petal is a convolution surface with a 2D cellular complex as a skeleton. An attaching surface of a petal is shown in Fig. 13. As mentioned in 4, the petals $\sqcup_i \mathcal{B}^3_{\text{petal}, i}$ ($i=0, 1, 2, \dots, 4$) are detached and have fallen after a while that is illustrated in

Fig. 14. The images in this case study have been generated without preliminary object conversion to polygonal meshes by direct ray-tracing of the FRep objects using a HyperFun plug-in to the POVray ray-tracer (www.povray.org).



Fig. 12 A geometrical model of flower with five petals $\sqcup_i \mathcal{B}^3_{\text{petal}, i}$ ($i=0, 1, \dots, 4$) attached to the receptacle $\mathcal{B}^3_{\text{receptacle}}$.



Fig. 13 The attaching surface $\mathcal{B}^2_{\text{petal}, i}$ of the i -th petal $\mathcal{B}^3_{\text{petal}, i}$.

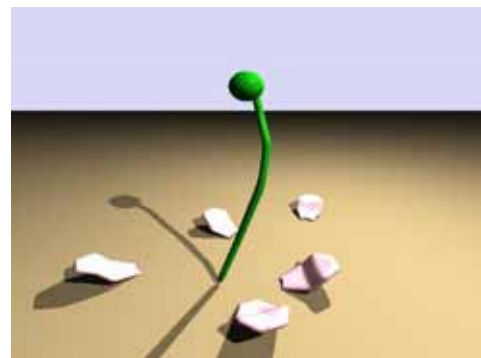


Fig. 14 The petals $\sqcup_i \mathcal{B}^3_{\text{petal}, i}$ ($i=0, 1, 2, \dots, 4$) have fallen on the ground, detached from the receptacle.

7. Conclusions and Future Works

We have confirmed that the abstraction hierarchy presented exhibits clear incremental modularity in modeling conceptual multiresolution analysis. In analyzing versatile information on the Web to get benefit in ever expanding Web applications of multiresolution analysis of digital contents as seen in e-commerce, e-manufacturing, e-medicine and e-learning, the proposed model works to our advantage in understanding the nature of changes taking

place in the modeled objects. It is, however, very difficult were we to depend on the traditional multiresolution analysis that are usually based on hierarchies of geometrical shapes, either physical, logical or both. Yet, the research on conceptual multiresolution analysis is still in its infancy waiting for extensive further works.

In terms of the impact factors, one of the highest priority applications of the conceptual multiresolution analysis is cognizing the complexity of the inter-operability of information systems. Such inter-operability problem arises in e-government projects and mega corporation projects where different information systems of merging organizations need to be interfaced. The complexity of the interfaces becomes combinatorial in brutal force interfacing. The preliminary research of one of the authors (TLK) indicates the possibility of making the complexity linear by extending and elaborating the conceptual multiresolution analysis presented in this paper.

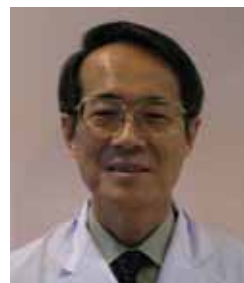
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Tosiyasu L. Kunii is currently Professor and IT Institute Director at Kanazawa Institute of Technology, Honorary Visiting Professor of University of Bradford, and Professor Emeritus of the University of Tokyo and of the University of Aizu. He was Professor of Hosei University from 1998 to 2003. Before that he served as the Founding President and Professor of the University of Aizu dedicated to computer science and engineering as a meta discipline, from 1993 to 1997. He had been Professor of Department of Computer and Information Science at the University of Tokyo from June 1978

until March 1993, after serving as Associate Professor at Computer Centre of the University of Tokyo in October 1969. He was visiting professors at University of California at Berkeley in 1994 and University of Geneva in 1992. He received his B.Sc. in 1962, M.Sc. in 1964 and D.Sc. in 1967 all from the University of Tokyo. He received the 1998 Taylor L. Booth Education Award of IEEE Computer Society. He is Fellow of IEEE and IPSJ. He has published over 50 books and over 300 refereed papers in computer science. Dr. Kunii was Founder and Editor-in-Chief of The Visual Computer: An International Journal of Computer Graphics (Springer-Verlag)(1984-1999), and International Journal of Shape Modeling (World Scientific)(1994-1995), and was Associate Editor of IEEE Computer Graphics and Applications(1982-2002). He is Associate Editor-in-Chief of The Journal of Visualization and Computer Animation (John Wiley & Sons)(1990-) and on the Editorial Board of Information Systems Journal(1976-), and Information Sciences Journal (1983-).



Masumi Ibusuki was born in Kyoto in December 16, 1940, and is currently Director of Dame Dental Clinic. He graduated from Faculty of Dentistry of Tokyo Medical and Dental University in March 1967, and obtained Doctor of Dental Surgery (D.D.S.) in May, 1967 and Ph.D. (Doctor of Dental Science) in March 1977. He was Professor at Faculty of Dentistry of Showa University.



Galina I. Pasko is currently a Researcher of the IT Institute of Kanazawa Institute of Technology in Japan. She received her M.Sc. degree in 1983 from Moscow Engineering Physics Institute, Russia. Her research interests include shape modeling, animation, and cultural heritage preservation.



Alexander A. Pasko is currently a Professor of the Faculty of Computer and Information Sciences, Hosei University, Japan. He received his Ph.D. degree in 1988 from Moscow Engineering Physics Institute (MEPI), Russia. He was a senior scientist at MEPI from 1988 to 1992. He was an assistant professor at the Department of Computer Software, University of Aizu, Japan (from 1993 to 2000). His research interests include shape modeling, animation, and computer art. He is a member of ACM SIGGRAPH, Eurographics Association, and IEEE Computer Society.



Daisuke Terasaki received the B.E. degree in industrial and system engineering from Hosei University, Tokyo, Japan, in 2001. He is currently a graduate student at the Graduate School of Computer and Information Sciences, Tokyo, Japan. His research interests include image recognition and intelligent computing.



Hiroshi Hanaizumi received the B.Sc. degree in communication engineering from the University of Electro-Communications, Tokyo, Japan, in 1978, and the M.Sc. and Dr. Eng. Degrees in instrumentation physics from the University of Tokyo, Japan, in 1980 and 1987, respectively. From 1981 to 1987 he was a research assistant at the Department of Mathematical Engineering and Information Physics, the University of Tokyo. He joined Hosei University in 1987 as a Lecturer in the Department of Industrial and Systems Engineering, was an Associate Professor from 1989

to 1995, and has been a Professor since 1996. Since 2000, he has been a member of the Faculty of Computer and Information Sciences, Hosei University. His current research areas include remote sensing, face recognition, and 3D medical image processing. He is a member of the IEEE Computer Society, the Geoscience and Remote Sensing Society, the Institute of Electronics, Information and Communication Engineers, and the Society of Instrument and Control Engineers.